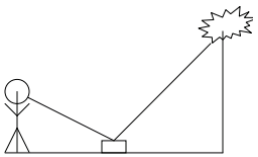
Curriculum Sample Template

Grade Level	10th	**Content Area**	Mathematics
Course Title (grades 9-12 Only)	Integrated Math II		
Expected Prior Knowledge *The knowledge/skills mastered earlier in the year foundational to the mastery of the required Standard in the current sample.*	Students understand ratio as a measure of the relative size of two quantities. Students can use and apply the Pythagorean Theorem to right triangles. Students also have a conception of angle measure as the openness of an arc and can envision a measure of that angle.		
Alignment to Program of Instruction *Briefly outline methods of instruction found in this sequence of lessons that align to the Program of Instruction described in A.3.*	This sample also demonstrates the three distinctive features of SySTEM Phoenix's Program of instruction: 1) Integration: Math and Science standards are taught in a mutually reinforcing manner 2) Problem-Based Learning: The standard is introduced to students within the context of a real-world problem that they continually hook their new knowledge into in order to create a viable solution or make an evaluative judgment 3) All final products are authentic documents that students will present to an audience of stakeholders.		
Standard Number and Description *List the Number* and the full description for the Required Standard or Science Performance Objective listed in section A.6.*	G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		
Materials/Resources Needed	Case Materials: Ten Things That You Will Not Find in Arizona. http://phoenix.about.com/od/phoenixfactsandfiction/a/wontfind.htm Her Secret is Patience. http://www.downtownphoenix.com/blog/2010/03/her-secret-is-patience/ Resources: "Sine and Cosine Activity A and B" Gizmo http://www.explorelearning.com Geogebra http://www.geogebra.org/cms/en/ "How to Make a Clinometer" http://www.wikihow.com/Make-a-Clinometer "Developing a Trigonometric Table" "Tangent, Cosine, and Sine…Oh, So Fine!"		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Standard (e.g., 6.EE.7). For HS Math, use Conceptual Category-Domain, Standard (e.g., A-REI.6). For Science, use Grade, Strand, Concept, PO (e.g., 4.1.3.1).*

Lesson (add as needed)	**Instructional Strategies**	**Student Activities**
1	***1. Essential Question:*** How do we calculate distances that can't be measured directly? ***2. Launch/Introduction:*** Show a picture of the "Her Secrete is Patience" installation. **The Case:** The Downtown Voices Coalition is concerned about the city of Phoenix's image. A recent article entitled "Ten Things That You Will Not Find in Phoenix Arizona. Landmarks and Skyscrapers? Nope." highlighted some of Phoenix's shortcomings when compared to major cities around the country. You will design a new addition to the Phoenix landscape that ameliorates at least one of the ten issues cited in the article. The Downtown Voices Coalition will only consider plans that are drawn to scale and provide an exact representation of how the design will look if commissioned. ***3. Vocabulary:*** Student Glossary Graphic Organizer of Tier 2 academic) and Tier 3 (content) vocabulary. "Term, Definition, Example." S***imilar triangles, ratio, right triangle.*** ***6. Investigate/Explore:*** Pythagoras and the Pyramid. Teacher will share the story of how Pythagoras won the respect of the Egyptians by measuring the height of their pyramids with only a stick and two shadows. ***8. Summarize/Debrief:*** After students have recreated the scenario and found similarity amongst the triangles, a representative from each group will share their findings and offer their explanation for Pythagoras' method. As a facilitator, the teacher will probe students at the whole class level to *construct viable arguments and critique the reasoning of others* and *look for and express regularity in repeated reasoning.* The teacher will also ensure the following learning targets are discussed: *1) students recognize similar triangles have corresponding proportional sides and corresponding congruent angles, 2) students can name and label the sides of right triangles as related to an acute angle (leg, hypotenuse, opposite, adjacent).*	***4.*** Students will be organized into collaborative working groups of four, which will stay consistent throughout the entire case. One student will have the role of group *facilitator* and a second student will have the role of *scribe. S*tudents will examine the problem and fill out first impressions using a graphic organizer to record answers to the following prompts: "Know, Need to Know, Hypothesis, Action Plan" OR 1.What is the problem?, 2.What do we know about the problem?, 3. What information or supplies do we need to solve the problem?, 4.What should we do to solve the problem? When finished, all groups will share out their initial responses in a whole class format. ***5.*** Then, the group will closely read the case material. The group facilitator will prompt group members to once again identify the problem. Once the problem is agreed upon, students will engage in a brainstorming protocol. The students present their association and ideas about the problem to find out what is already known and how does the case relate to their previous knowledge. The ideas are said aloud and written on self-stick notes, which are organized by the group. The scribe records all idea clusters in the graphic organizer for future reference. ***7.*** Students will carefully draw a depiction of Pythagoras' measurement method. Groups will be encouraged to use items around the classroom to reconstruct the scenario. Students will measure all aspects of the two triangles using a ruler and a protractor. Students will record their observations and attempt to describe how Pythagoras calculated a distance he could not measure directly. ***9. Formative Assessment:*** Three problems require students to determine the measurement of an unknown object given two shadows and a reference object. ***10. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct an initial version of the explanation for the problem and identify the most important concepts and their relationships. The group may delegate individual group members to gather specific information for the group.
2	***1. Introduction:*** Teacher will prompt students to reflect on why	***2. Investigate/Explore:*** Given the measurements of 8 different right

	different fractions represent the same ratio $\left(\text{ie. } \frac{1}{2}, \frac{2}{4}, \frac{3}{6}\right)$. ***3. Summarize/Debrief:*** As a whole group, students will *construct viable arguments and critique the reasoning of others.* Facilitator will ensure the following learning target is formalized: *students use proportions to determine side lengths of similar right triangles.* ***7. Summarize/Debrief:*** As a whole group, students will *look for and express regularity in repeated reasoning* to construct equations to represent the trigonometric functions. Facilitator will ensure the following learning target is formalized: *students understand that the similarity in right triangles leads to proportional relationships which produce the trigonometric ratios of the acute angles in the right triangle.* Challenge students to create a mnemonic for SOH-CAH-TOA.	triangles, students will work in groups to determine which are similar. ***4. Formative Assessment:*** Each student will create two index cards, an example and non-example of a pair of similar triangles. On the back, students will defend their answers using proportional reasoning. Students will partner up and use the "quiz-quiz-trade" protocol. ***5. Investigate/Explore:*** Students will use gizmos from elearning.com to interactively explore trigonometric ratios of right triangles using Sine and Cosine Activity A and B. The facilitator will ensure students observe the following: *1. The sin ratio of an acute angle changes when the opposite leg is modified but not when the adjacent leg is modified, 2. The cosine ratio of an acute angle changes when the adjacent leg is modified but not when the opposite leg is modified, 3. Neither the sin nor cosine ratio changes when the scale of a right triangle is modified.* ***6. Reflection:*** Why do similar triangles maintain the same trigonometric ratios? Explain why the sine of x is the same regardless of which triangle is used to find it in the figure. ***8. Formative Assessment:*** Three problems require students to determine the measurement of an unknown side of a special right triangle given one angle and one length. Extension question will ask students to describe a process for measuring any given building using household materials. ***10. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct an initial version of the explanation for the problem and identify the most important concepts and their relationships. The group may delegate individual group members to gather specific information for the group.
3	***1. Introduction:*** Landscape designers know that measurement is crucial to creating a replicable scale model. One mere inch can be the difference between a working bridge and a fatal disaster. Discuss the need to *attend to precision.* ***2. Modeling:*** Today, each group will use what they have learned about similar triangles and trigonometric ratios to determine the height of various "SySTEM Metro" landmarks. Teacher will demo how to measure the angle of approach and departure when a laser light is reflected into a mirror. ***5. Summarize/Debrief:*** As a whole group, each student will share	***3. Investigate/Explore:*** Students will measure the angle of approach and departure by reflecting a laser light into a mirror, tracing its path on paper, and measuring the angles. The teacher will ensure students understand that the angle of approach and departure will always be congruent. Students will then use the diagram below to label the measurements and show the proportions that allow the student to determine the missing height. Facilitator will ensure students identify the corresponding parts that make the two right triangles similar.

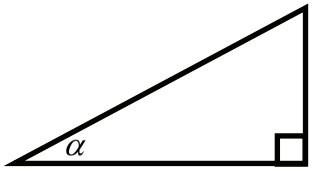
	his/her height measurements for each landmark. Facilitator will select students with different calculation approaches measurements and/or different to present their reasoning to the class. *Students will critique the reasoning of others.* Facilitator will ensure the following learning target is formalized: *student can recognize that if two right triangles have a pair of acute congruent angle, that the triangles are similar.*	 ***4. Formative Assessment:*** Students will collect data on a series of landmarks using the laser/mirror method. Height calculations will be done upon return to class. ***5. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct an initial version of the explanation for the problem and identify the most important concepts and their relationships. The group may delegate individual group members to gather specific information for the group.
4	***1. Introduction:*** A trigonometry table is a powerful tool used by mathematicians and was first developed by Hipparchus who lived in the 2nd century BC. Today, you will create a trigonometry table. ***3. Summarize/Debrief:*** Using a "Jigsaw" protocal, the original teams will create new teams with members from different groups. Teams will first share their answers to questions 1-4 and seek consensus. Then, each member will present its Case Work problem to the group and receive feedback. The teacher as facilitator will ensure the following learning target is formalized: s*tudent can recognize the special right triangle relationships of the 30-60-90 and 45-45-90 triangles.*	***2. Investigate/Explore:*** Students will use special triangles to develop the concept of trigonometric ratios. Each student will create a trigonometry table with the procedure detailed in "Developing A Trigonometric Table". After the table is complete, students will answer the following questions as a group: 1. What pattern do you see for sine, cosine, and tangent as the angle increases (3 patterns total)?, 2. For each of the three patterns noticed, explain why this pattern occurs using your knowledge of trigonometric ratios., 3. What pattern do you see for sine, cosine and tangent as the angel decreases?, 4. For each pattern, explain why it occurs using your knowledge of trigonometric ratios., 5. Write your own real-world problem involves a right triangle and relates to your Case Work., 6. Solve your problem using trigonometry. ***4. Investigate/Explore:*** Research "How to Make a Clinometer". After building the apparatus, use it to calculate the heights of measurable objects in the "Tangent, Cosine, and Sine…Oh, So Fine!" inquiry lab. ***5. Formative Assessment:*** Set the clinometer to 45°, and measure the height of the east wall. What special triangle relationships will facilitate your measurement? If the clinometer was set to 30°, what special triangle relationships would facilitate your measurement? Extension question will ask students to discuss under what circumstances a larger clinometer angle would be more appropriate for measuring than a smaller one.

	1. Introduction: Teacher will review the peer review schedule and product expectations for the PBL case. Teacher will review rubric and deadlines for self-assessment. ***3. Group Conferencing/Feedback:*** As students conduct the peer review protocol, the teacher may observe or participate in group dialogue. The teacher will actively observe groups, help students with group dynamic issues if needed, and ask probing questions. ***5. Direct Instruction/Workshop:*** Teacher will workshop the IMRAD (introduction, methodology, results, analysis, and discussion) structure to groups and/or members who choose to attend.	2. ***Group Conferencing/Feedback:*** Two student groups will pair up to share their design plans. Using a peer review protocol, the original group will share their process, findings, and solution and look for critical feedback and suggestions. Then, the groups will switch roles. ***4. PBL Case:*** Student groups will create a presentation to share their solution to the PBL Case. The presentation will be structured in the IMRAD (introduction, methodology, results, analysis, and discussion) format. Students will send their work to a member of the Downtown Voices Coalition for review. ***6. Closure/Self-Assessment:*** Individually, each student will self-assess their learning through a written summary which details how their individual contributions furthered the group's effort in addition to a reflection of learning. Students will use the claim-evidence-reason framework to support their position.
	3. Throughout the revision/completion process, the role of the teacher is facilitative. The teacher will actively observe groups, help students with group dynamic issues if needed, ask probing questions and/or direct towards further resources.	***1. Summative Assessment:*** Students will take summative assessment. 2. Students refine their design based on feedback from Downtown Voices Coalition. Student groups will continue to refine their presentation and may videotape simulations. 4. ***Closure/Group-assessment:*** Groups will meet for group self-assessment and score their presentation on final rubric. Each score must include claim-evidence-reason framework.
	1. Presentation/Participation: Final presentation cycle for each group to include: a) Group solution presentation, b) Open forum clarifying questions , c) Open forum probing questions, d) Community member feedback ***3. Summarize/Debrief:*** After all presentations are finished, teacher will facilitate a debrief of the problem.	2. As groups present, all audience members, teachers, students, and community members will write down key points and questions. All participants will have the opportunity to ask follow up questions after each presentation. ***4. Closure:*** After group debrief, students will incorporate synthesis of learning experience into individual self-assessment.

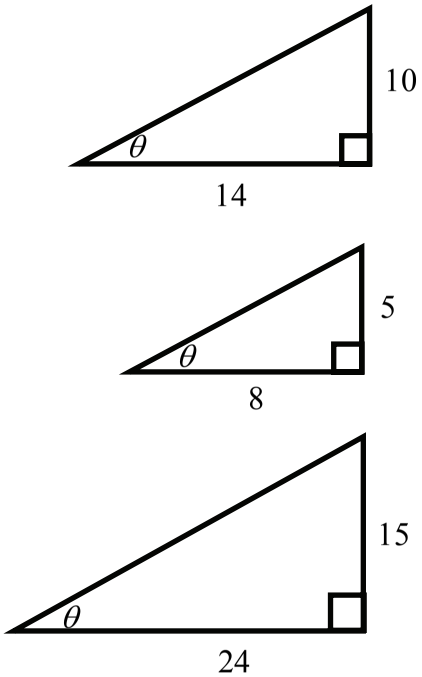
Summative Assessment Items: *Attach here.* For each summative assessment item, include an answer key or scoring rubric describing how points are to be awarded.

Type I:

1. Given the acute angle α, label the following sides: Opposite, Adjacent, Hypotenuse



2. Determine which of the three triangles below are similar. Provide an explanation and proof for your answer.



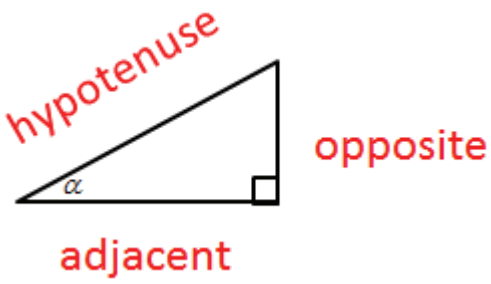
SOLUTION

Scoring: This item represents a level 2 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can name the sides of right triangles as related to an acute angle.

Student can determine the similarity of two triangles by verifying the ratio of two sides in one triangle is equal to the corresponding two sides of another triangle.

1.

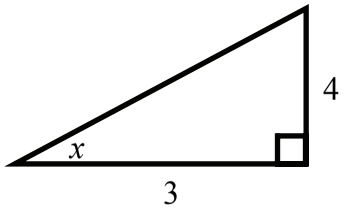


2.
The second and third triangles are similar, because similar triangles have proportional corresponding sides.

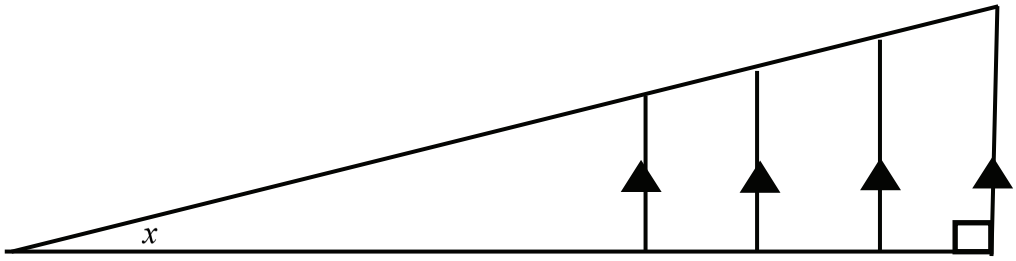
$$\frac{\text{opposite}}{\text{adjacent}}: \frac{10}{14} \neq \frac{5}{8} = \frac{15}{24}$$

Type II

1. Find the sin, cosine, and tangent of x.



2. Explain why the sine of x is the same regardless of which triangle is used to find it in the figure.



SOLUTION

1. $\sin x = \frac{4}{5}$

   $\cos x = \frac{3}{5}$

   $\tan x = \frac{4}{3}$

Scoring: This item represents a level 3 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

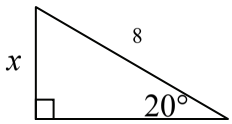
Student can understand that the ratio of two sides in one triangle is equal to the ratio of the corresponding two sides of all other similar triangles.

Student can define sine, cosine, and tangent as the ratio of sides in a right triangle.

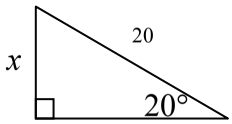
2. The angles of every triangle represented in the diagram are all similar because each side opposite to the angle is parallel and thus maintains its corresponding angles. Because $\sin x$ is the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ and the ratio of corresponding sides amongst similar triangles are the same, $\sin x$ will be the same regardless of which triangle is used.

Type III

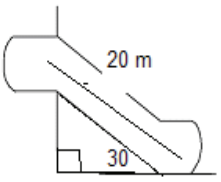
1. Given $\sin 20^\circ = 0.342$ find the value of x in the diagram, correct to 1 decimal place.



2. Given $\sin 20^\circ = 0.342$ find the value of x in the diagram, correct to 1 decimal place. Then, explain how problem #2 and #3 demonstrate that trigonometric ratios are a property of an acute angle.



3. An escalator at an airport slopes at an angle of 30° and is 20 m long. Through what height would a person be lifted by travelling on the escalator? Use proportional reasoning to determine the answer.

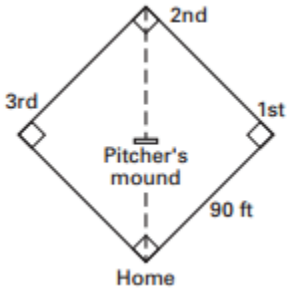


4. The infield of a baseball field is a square. The distance from home plate to first base is 90 feet.

A. What is the distance from home plate to second base?

B. What is the distance from third base to first base?

C. If the pitcher's mound is 60 feet 6 inches from home plate, is it the midpoint of the diagonal from home plate to second base?



Scoring: This item represents a level 4 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can compare common ratios for similar right triangles and develop a relationship between the ratio and the acute angle leading to the trigonometry ratios.

Student can calculate unknown sides of special right triangles.

1.

$$\sin\theta = \frac{opp}{hyp} \rightarrow \sin 20 = \frac{x}{8} \rightarrow 0.342 = \frac{x}{8} \rightarrow x \approx 2.7$$

2.

$$\sin\theta = \frac{opp}{hyp} \rightarrow \sin 20 = \frac{x}{20} \rightarrow 0.342 = \frac{x}{20} \rightarrow x \approx 6.8$$

Trigonometric ratios are a property of the angle because the ratio of the side opposite of the angle θ to the hypotenuse is constant for a given angle θ and does not depend on the size of the right triangle. $\frac{opp}{hyp} \approx \frac{2.7}{8} \approx \frac{6.8}{20} \approx .34$

3. For in a 30-60-90 triangle, maintain a proportion of $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$ or $1x, x\sqrt{3}, 2x$ respectively. Since the hypotenuse, $2x$, is 20, $x = 10$, and the height, x, must be 10.

4. A. $90\sqrt{2}$ B. $90\sqrt{2}$ C. The midpoint would be $45\sqrt{2} \approx 63.6$. So, no the home plate is not located at the midpoint.