Curriculum Sample Template

Grade Level	11	**Content Area**	Math
Course Title (grades 9-12 Only)	Integrated Math III		
Expected Prior Knowledge *The knowledge/skills mastered earlier in the year foundational to the mastery of the required Standard in the current sample.*	Students can use the properties and laws of exponents to transform expressions for exponential functions as well as solve exponential equations, including in real world contexts. Students recognize an exponential function as a rule for applying growth or decay factors and can graph these functions from data and equations in the form $y = ab^x$ and its contextualized variants such as $A = P(1+r)^t$ and $A = Pe^{rt}$		
Alignment to Program of Instruction *Briefly outline methods of instruction found in this sequence of lessons that align to the Program of Instruction described in A.3.*	This sample also demonstrates the three distinctive features of SySTEM Phoenix's Program of instruction: 1) Integration: Math and Science standards are taught in a mutually reinforcing manner 2) Problem-Based Learning: The standard is introduced to students within the context of a real-world problem that they continually hook their new knowledge into in order to create a viable solution or make an evaluative judgment 3) All final products are authentic documents that students will present to an audience of stakeholders.		
Standard Number and Description *List the Number* and the full description for the Required Standard or Science Performance Objective listed in section A.6.*	F-LE.4 For exponential models, express as a logarithm the solution to ab^(ct)=d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.		
Materials/Resources Needed	"Modeling Exponential Growth" http://www.esrl.noaa.gov/gmd/infodata/lesson_plans/Modeling%20Exponential%20Growth.pdf Graphing Utilities: Free TI-84, Scientific Functions Library www.classzone.com, "Logarithms Demystified" NCTM Planets, Sun and Star Sizes compared http://www.youtube.com/watch?v=Fj_PjkQqrDo "Logarithmic Functions Activity A and B" http://www.explorelearning.com		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Standard (e.g., 6.EE.7). For HS Math, use Conceptual Category-Domain, Standard (e.g., A-REI.6). For Science, use Grade, Strand, Concept, PO (e.g., 4.1.3.1).*

Lesson (add as needed)	Instructional Strategies	Student Activities
1	***1. Essential Question:*** How can a best fit function be determined for a set of data? ***2. Launch/Introduction:*** You are the senior quality assurance manager at Johnson Utilities. Your team focuses on the operation and maintenance of water treatment facilities, including sewage treatment plant water wells and booster stations. Last year, the presence of E. Coli in the San Tan Valley's water supply made the news and negatively impacted the company's public image. Your task is to design a schedule and implementation plan for all water quality sample activities within the company. The challenge is to include the necessary mathematical models and evidence to prove this new plan will not allow E. Coli to reach critical contamination levels. Your audience is a group of community members of the San Tan valley. These individuals will decide if your plan is secure enough to reestablish trust in the ability of Johnson Utilities to provide safe, healthy drinking water. ***3. Vocabulary:*** Student Glossary Graphic Organizer of Tier 2 academic) and Tier 3 (content) vocabulary. "Term, Definition, Example" Logarithm. ***7. Instruction:*** The teacher will write $y = b^x$ on the board and prompt students to identify what each variable represents (*b* is the base, *x* is the exponent, *y* is the power). Teacher will pose the question: "If the inverse of an exponent is a root, what is the inverse of a base?" (logarithm) Diagram an exponential equation and compare to logarithmic equation. In general, if $y = b^x$, then $x = \log_b(y)$. In short, a logarithm asks the question "what's the exponent?" ***9. Summarize/Debrief:*** As a whole group, students will *look for and express regularity in repeated reasoning* to generalize the relationship between an exponent and logarithm with the same base. Facilitator will ensure the following learning target is formalized: *1) student can convert exponential equations to logarithmic equations and 2) student can convert logarithmic equations to exponential equations.*	***4.*** Students will be organized into collaborative working groups of four, which will stay consistent throughout the entire case. One student will have the role of group *facilitator* and a second student will have the role of *scribe. S*tudents will examine the problem and fill out first impressions using a graphic organizer to record answers to the following prompts: "Know, Need to Know, Hypothesis, Action Plan" OR 1.What is the problem?, 2.What do we know about the problem?, 3. What information or supplies do we need to solve the problem?, 4.What should we do to solve the problem? When finished, all groups will share out their initial responses in a whole class format. ***5. Investigate/Explore:*** Students will use gizmos from elearning.com to interactively compare the equation of a logarithmic function to its graph and its associated exponential equation/graph in Logarithmic Functions Activity A and B. The facilitator will ensure students observe the following: *Logarithmic and exponential functions with the same base are inverse functions.* *6.* ***Accessing Prior Knowledge:*** In their teams, each student will have two minutes to write as much as possible about the following prompt: "What are inverse operations? List all the inverse operation pairs you know". As a whole group, students will create a comprehensive list. 8. ***Investigate/Explore:*** Students will *look for and make use of structure* to justify the following properties of logarithms: $\log_a 1 = 0$, $\log_a a = 1$, $\log_a a^x = x$, and $\log_a x = \log_a y$, then $x = y$. ***10. Reflection/Closure:*** Most logarithmic calculations are done in base 2,10, or *e*. Why would these particular bases be preferred? ***11. Formative Assessment:*** Given several equations in logarithmic form, students will convert to exponential form (and vice versa).

2	**1. *Introduction:*** Before calculators and computers, people still needed to calculate with very large and very small numbers. Logarithms assisted with this process and today you will learn how. ***3. Debrief/Summarize:*** As a whole group, students will c*onstruct viable arguments and critique the reasoning of others* as students connect how the laws of logarithms convert multiplication into addition, division into subtraction, and exponentiation into multiplication. A graphic organizer table with headers "operation, exponent law, logarithm law" will provide a reference for students to relate the laws to logarithms to those of exponents. Facilitator will ensure the following learning target is formalized: *student can apply properties of logarithms to simplify log expressions.*	**2. *Stimulus:*** Students read "History of logarithms." Students will explore the "shortcut" provided by logarithms and how the logarithmic scale is used to express both very small and very large measurements. ***3. Investigate/Explore:*** Using NCTM "logarithms demystified" students will use a simplified slide rule to discover the properties of logarithms. Students will *look for and express regularity in repeated reasoning.* ***5. Reflection:*** Students will respond to the prompt: "How are the properties of exponents and the properties of logarithms related?" ***4. Formative Assessment:*** First, students will watch the clip "Planet, Sun, and Star Sizes compared." Given the volume of objects in the solar system in cubic kilometers, students will use the properties of logarithms to calculate by hand the answer to questions such as "How many moons could fit in Earth?" and "How many Earths could fit in the sun? Jupiter?". ***5. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct an initial version of the explanation for the problem and identify the most important concepts and their relationships. The group may delegate individual group members to gather specific information for the group.
3	***2. Modeling:*** Teacher will present and "think aloud" the methodology to explore Illustrative Math's problem 784 "Accuracy of Carbon 14 Dating II." The problem bridges students prior understanding of exponential modeling, while requiring a logarithmic answer due to the exponential variable in the function. ***4. Summarize/Debrief:*** As a whole group, each group will have a representative share their findings. Facilitator will select students with different approaches and/or answers to present their reasoning to the class. *Students will critique the reasoning of others.* Facilitator will ensure the following learning targets are formalized: *1) for exponential models, student can express as a logarithm the solution to* $ab^{ct} = d$ *where a, c, and d are numbers and the base b is 2, 10, or e and 2) student can use technology to evaluate a logarithm.* 6. Throughout the model construction process, the role of the teacher is facilitative. The teacher will actively observe groups, help students with group dynamic issues if needed, ask probing questions and/or	***1. Stimulus:*** Human Memory Model – Students were given an exam and then tested monthly with an equivalent exam. The average score for the class was given by the human memory model $f(t) = 80 - 17\log_{10}(t+12), \quad 0 \le t \le 12$, where *t*, is time in months. What is the average score on the original exam? after 4 months? 10 months? Verify your answers using a graphing utility. The teacher will observe and collect data on the following learning target: *student can use technology to evaluate a logarithm.* ***3. Investigate/Explore:*** Presented in the scenario in Illustrative Math's problem 382 "Newton's Law of Cooling", students will *look for and make use of structure* to explain how the equation $f(x) = 110e^{-0.08t} + 75$ models the scenario. Students will then express the solution as a variable. *Differentiation:* Students may go on to further models such as the population growth (easier) and/or exponential decay (difficult). *Challenge:* Encourage students to solve the model for *t* using logarithms to graph the entire solution set.

	direct towards further resources.	***5. PBL Casework:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct a model of the E. Coli growth and to test this model against their research thus far. The group may delegate individual group members to gather specific information for the group.
4	**1.** ***Introduction:*** Probably the most significant factor in causing the resource and environmental predicament that we find ourselves in is the phenomenon of exponential growth. Over the past few decades, population, nonrenewable resource consumption, production, industrial output and pollution generation have all increased exponentially. Today we will use the properties of logarithms to make sense of exponential models. ***3. Modeling:*** Teacher will demonstrate how to input data into a graphing utility (resources listed) and generate an exponential regression. As an equation, the exponential regression can be used to solve, in this case, for the particular population in any year. To solve for a variable exponent, the teacher utilizes the inverse relationship of logarithms and the base of an exponent. *6.* ***Group Conferencing/Feedback:*** As students conduct the peer review protocol, the teacher may observe or participate in group dialogue. ***7.*** Teacher will review rubric and deadlines for self-assessment.	**2.** ***Stimulus:*** Students will use the activity "Modeling Exponential Growth" to conduct an exponential growth simulation and collect data using dice for three scenarios: Unrestricted Exponential Growth, Limited Birth Control Program, and Zero Population Growth Plan. Students will *look for and express regularity in repeated reasoning*. ***4. Formative Assessment:*** Given data from a real world situation (i.e. rabbit population), students will generate an exponential regression and use the equation to predict future population levels. ***5. PBL Casework:*** ***8. Closure/Self-Assessment:*** Individually, each student will self-assess their learning through a written summary which details how their individual contributions furthered the group's effort in addition to a reflection of learning. Students will use the claim-evidence-reason framework to support their position.
4	3. Throughout the revision/completion process, the role of the teacher is facilitative. The teacher will actively observe groups, help students with group dynamic issues if needed, ask probing questions and/or direct towards further resources.	***1. Summative Assessment:*** Students will take summative assessment. ***2. PBL Casework:*** Students refine their presentation based on feedback and self-assessment information. Student groups will continue to refine their presentation. 4. ***Closure/Group-assessment:*** Groups will meet for group self-assessment and score their presentation on final rubric. Each score must include claim-evidence-reason framework.
5	***1. Presentation/Participation:*** Final presentation cycle for each group to include: a) Group solution presentation, b) Open forum clarifying questions , c) Open forum probing questions, d) Community member feedback ***3. Summarize/Debrief:*** After all presentations are finished, teacher will facilitate a debrief of the problem.	2. Groups will present. While listening to the presentation, teachers, students, and community members will write down key points and questions. All participants will have the opportunity to ask follow up questions after each presentation. ***4. Closure:*** After group debrief, students will incorporate synthesis of learning experience into individual self-assessment

Summative Assessment Items: *Attach here.* For each summative assessment item, include an answer key or scoring rubric describing how points are to be awarded.

Type I:

1. Given the values of the powers of 2 (i.e. $2^0 = 1,\ 2^1 = 2,\ 2^2 = 4,\ 2^3 = 8, \ldots$) estimate the value of $\log_2 73$ **without** the use of technology.

2. Using technology, find the value of $\log_2 -4$. Using the properties of exponents, explain the result.

3. Expand using logarithmic properties $\ln \frac{3x^2}{y+1}$

4. Convert $\log_2 \frac{1}{16} = -4$ to exponential form.

5. Rewrite the following in logarithmic form.

$4^5 = 1024$

SOLUTION

Scoring: This item represents a level 2 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can apply properties of logarithms to simplify log expressions.

Student can use technology to evaluate a logarithm.

Student can convert exponential equations to logarithmic equations.

1. The log of 73 will be between 6 and 7 since $2^6 = 64$ and $2^7 = 128$. Since 73 is much closer to 64, $\log_2 73 \approx 6.1$.

2. When put in a calculator, $\log_2 -4$ yielded the response "Error – non-real result". A log is the inverse operation of a base. In this case, the question asked is 2 to what power yields -4. Since a base 2 to any real exponent will never yield a negative result, there is no real solution.

3. $\ln 3 + 2\ln x - \ln(y+1)$

4. $2^{-4} = \frac{1}{16}$

5. $\log_4 1024 = 5$

Type II:

1. The number of bacteria present in a culture after t minutes is given as $B = 1000e^{kt}$, where k is a constant. If there are 8,520 bacteria present after 15 minutes, find k.

2. A biologist is culturing a new strain of bacteria, starting with just a few cells. When she first observes the bacteria colony, the population is 1,000 cells and is doubling every 20 minutes. This rate of growth continues for the rest of the day.

A. Write a function $P(t)$ that represents the population of the bacteria colony as a function of time in minutes.

B. How long will it take the population to reach 8,000?

C. How long will it take the population to reach 20,000? Give an exact answer in terms of logarithms.

SOLUTION

Scoring: This item represents a level 3 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

For exponential models, student can express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e.

1. $k \approx 0.14$

2. A. $P(t) = 1000\left(2^{\frac{t}{20}}\right)$ or $P(t) = 1000e^{\left(\left(\ln\frac{2}{20}\right)t\right)}$

B. 60 minutes / 1 hour

C. $t = 20\log_2 20$ or 20(ln20/ln2)

Type III:

Use the estimates of the world population from 10,000 BCE to 2010 CE to perform the tasks below.

Year	Population (in Millions)
10000 BCE	1
8000 BCE	5
6000 BCE	5
4000 BCE	7
2000 BCE	27
1 AD	170
200 AD	190
400 AD	190
600 AD	200
800 AD	220
1000 AD	254
1200 CE	360
1400 CE	350
1600 CE	545
1800 CE	813
1950 CE	2400
1960 CE	3000
1970 CE	3800
1980 CE	4500
1990 CE	5300
2010 CE	6900

Source: *data from U.S. Census Bureau's International Data Base (10,000 BCE-1950 CE, 1950 CE-2010 CE)*

A. Create a mathematical model of the population growth from 10,000 BCE to 1950 CE using the exponential regression.

B. Use the model to approximate the population in 2010 CE. How does the model compare to the actual population in 2010 CE?

C. Use the model to estimate the population in 2020 CE.

D. Repeat parts A through C using population data from 1950 CE to 2010 CE.

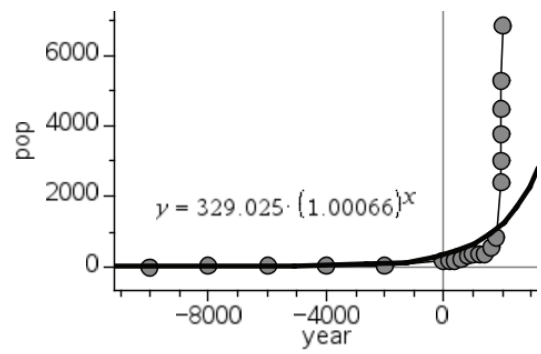
E. Discuss the advantages and disadvantages of each model.

SOLUTION

Scoring: This item represents a level 4 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Create an exponential model for a set of real world data and defend its design.

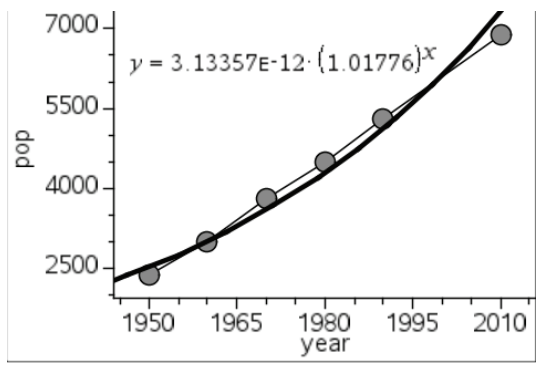
A.



B. The population was approximately 1246.79 million people in 2010. The model is not close to actual 2010 data of 6900 million people.

C. The population will reach approximately 1255.08 million people by 2020.

D.
a.



b. The population was approximately 7379.31 million people in 2010.
c. The population was approximately 8714 million people in 2020.

E. Though the first model includes more data points, the tremendous growth experienced in the 21st century is not represented due to the influence of the slow growth trends of centuries prior. The second model provides a much closer approximation to the data, however, may grow too quickly to provide an accurate model for future projections