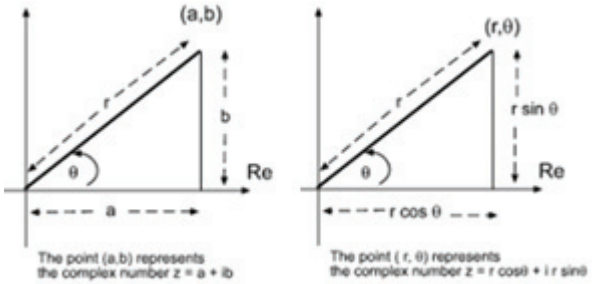
Curriculum Sample Template

Grade Level	High School	**Content Area**	Math
Course Title (grades 9-12 Only)	Calculus		
Expected Prior Knowledge *The knowledge/skills mastered earlier in the year foundational to the mastery of the required Standard in the current sample.*	Students are adept at the algebraic treatment of complex numbers but have not worked with polar coordinates on the complex plane. Students can work algebraically with complex numbers. Students are able to construct trigonometric identities on the unit circle.		
Alignment to Program of Instruction *Briefly outline methods of instruction found in this sequence of lessons that align to the Program of Instruction described in A.3.*	This sample also demonstrates the three distinctive features of SySTEM Phoenix's Program of instruction: 1) Integration: Math and Science standards are taught in a mutually reinforcing manner 2) Problem-Based Learning: The standard is introduced to students within the context of a real-world problem that they continually hook their new knowledge into in order to create a viable solution or make an evaluative judgment 3) All final products are authentic documents that students will present to an audience of stakeholders.		
Standard Number and Description *List the Number* and the full description for the Required Standard or Science Performance Objective listed in section A.6.*	N-CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.		
Materials/Resources Needed	Documents: "Coordinate Cornhole.doc", "Real-Life Applications of Polar Coordinates.doc" Website Resources: Plot Points in Polar Coordinates http://www.analyzemath.com/polarcoordinates/plot_polar_coordinates.html Polar Coordinates "The Basics" "Basic Graphing" http://www.justmathtutoring.com/ Polar Coordinates and Complex Numbers Module http://www.wmich.edu/math/kaap/modules/polar-complex/polar-complex.swf http://www.onlinemathlearning.com/complex-numbers-convert.html Graphing Software: Electronic Function Library www.classzone.com TI-84 Emulator http://www.bcs1.org/webpages/hsmath/ti-84.cfm		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Standard (e.g., 6.EE.7). For HS Math, use Conceptual Category-Domain, Standard (e.g., A-REI.6). For Science, use Grade, Strand, Concept, PO (e.g., 4.1.3.1).*

Lesson (add as needed)	**Instructional Strategies**	**Student Activities**

1	***1. Essential Question:*** How are the different representations of a polynomial function (or its solutions) related?
	
	2. Launch/Intro: **The Case:** You are employed by Cox Communications. Your team has designed a revolutionary satellite system that will take TV, Internet, Cell Phone, and GPS communication leaps and bounds into the future. After many months of research and development, your team is ready to launch four satellites into space to make your communications system available to the world. The computer programmers are waiting on the orbital path information so they may program the satellites. In order to avoid the fate the Iridium 33 and Cosmos 2250 faced, you are seeking to choose a path that minimizes the potential of collision with other orbiting bodies in space. Cox Communications requires each team to choose the same orbital plane. Each team member must name his/her satellite, graph the path in the orbital plane, and substantiate the safety of the satellite with a small margin of error. Individual satellite information is to be compiled in a team presentation that includes a digital or physical model. Details of the mathematical supporting data must be provided for those members of the board who wish to review the proof substantiating your claims.
	
	3. Vocabulary: Student Glossary Graphic Organizer of Tier 2 academic) and Tier 3 (content) vocabulary. "Term, Definition, Example" Complex plane, rectangular form, polar form, modulus, argument
	
	5. Stimulus: Coordinates boil down to a set of directions. Depending on the situation, one may want to specify their location in different ways. Students will consider the difference between giving a friend directions to the mall house versus an alien directions to Earth. Consider this point (dot on the ceiling) and the kinds of directions you could give someone to locate it.
	
	7. Debrief/Summarize: Students discuss their observations. The teacher will ensure the following learning targets are formalized: *1) students can locate a polar coordinate by specifying the length, r, of the directed segment from the pole to the point, and the measure θ of the angle this segment makes with the polar axis and 2) students can*
		4. Students will be organized into collaborative working groups of four, which will stay consistent throughout the entire case. One student will have the role of group *facilitator* and a second student will have the role of *scribe. S*tudents will examine the problem and fill out first impressions using a graphic organizer to record answers to the following prompts: "Know, Need to Know, Hypothesis, Action Plan" OR 1.What is the problem?, 2.What do we know about the problem?, 3. What information or supplies do we need to solve the problem?, 4.What should we do to solve the problem? When finished, all groups will share out their initial responses in a whole class format.
		
		6. Investigate/Explore: Plot Points in Polar Coordinates website. Students will follow the interactive lab and make the following observations about coordinates in polar form: *polar coordinates consist of an ordered pair (r, θ)*, sometimes written as *(r,t)*.
		
		8. Formative Assessment: Students will play a game of Cornhole. Using a giant coordinate plan made of tape on the floor, students will aim for one of five targets. Students will individually estimate the angle and the distance of each point and record their answers onto a worksheet. One point will be awarded for each correct radius and one for each correct theta.
		
		9. Formative Assessment: Students will plot four specified polar coordinates on the polar plane. Students will also give the coordinates for four points graphed on a polar plane. Differentiation: Graph polar coordinates online at http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0.
		
		10. Reflection/Closure: Students will provide examples to support the following statements:
		• In rectangular coordinates
		  ○ Each set of coordinates represents a unique point
		  ○ Each point has a unique set of coordinates
		• In polar coordinates
		  ○ Each set of coordinates represents a unique point
		  ○ But, every point has many possible sets of polar

	explain that a point in a polar coordinate system has many different pairs of polar coordinates.	coordinates
2	***1. Introduction:*** Starting with a figure titled "The Complex Plane, Take Two" with a picture of a horse labeling the x-axis and a picture of a unicorn labeling the y-axis, students are encouraged to interpret the meaning. (Answer: The horse represents real numbers and the unicorn represents imaginary numbers). ***2. Modeling:*** Teacher will model how any complex number, $z = a + bi$, is graphed on the complex plane *a* units on the "real" or x-axis and *b* units on the "imaginary" or y-axis. Students will practice with several examples in a "think-pair-share" structure. Teacher will informally assess: *Student can graph a complex number in the complex plane in rectangular form.* ***5. Summarize/Debrief:*** A representative from each group will present their *abstract and quantitative reasoning* process to reconcile the equation. As a facilitator, the teacher will probe students at the whole class level to *look for and make use of structure* provided from the trigonometric ratios. For example: • If $\sin\theta = \frac{y}{r}$, then $y = r\sin\theta = b$. Therefore, $(r\sin\theta)i = r(i\sin\theta) = bi$. Lead students to see the relationship between (x, y) and (r, θ) The teacher will also ensure the following learning targets are formalized: 1) *student can graph a complex number in the complex plane in polar form, 2) students know and can explain (based on trigonometric identities) why both forms, rectangular and polar, represent the same number.*	***3. Accessing Prior Knowledge:*** In their teams, each student will have one minute to write as much as possible about the following prompt: "Write down everything you remember about trigonometry". Students will then pass their paper to their partner, read the response and add something new. This process will continue until each paper returns to the original writer. Teacher will collect data based on responses. As a whole group, students will create a comprehensive list. ***4. Investigate/Explore:*** Students will *look for and make use of structure* given the following prompt. Use the figures below to prove the following statement: $a + bi = r(\cos\theta + i\sin\theta)$  ***6. Formative Assessment:*** Students will write a paragraph proof, in which students prove a statement by writing successive sentences stating evidence that leads to a conclusion, to verify the statement "(a,b) and (r, θ) are equivalent ways to represent a complex number as a point in the complex plane." ***7. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. Teams will set new goals and conduct self-directed research.
3	***1. Introduction:*** Students will read "Real-Life Applications of Polar Coordinates." Students will respond to the information in their KWHLS chart and share ideas in a whole group format. 3. ***Summarize/Debrief:*** The facilitator supports students to master the following learning target: 1) *student can convert complex numbers from rectangular to polar form and vice versa on the complex plane,*	***2. Investigate/Explore:*** **S**tudents will l*ook for and make use of structure* when converting complex numbers from rectangular to polar form and vice versa. Students may logically reason the conversion or find tutorials on this skill from a variety of websites (listed in resource section).Students will use graphing software to verify that both coordinate types graph to exactly the same location.

	2) student can plot a complex number represented in rectangular form on the complex plane. *5.* ***Group Conferencing/Feedback:*** As students conduct the peer review protocol, the teacher may observe or participate in group dialogue. ***6.*** Teacher will review rubric and deadlines for self-assessment.	***4. Formative Assessment:*** Given a coordinate in one form, students will convert it into the other. For example, given a rectangular coordinate, students will a) graph the point on the complex plane, b) calculate its distance to the origin, *r*, c)calculate the angle between *r* and the *x*-axis, d) write the coordinate in polar form. ***5. PBL Casework:*** ***7. Closure/Self-Assessment:*** Individually, each student will self-assess their learning through a written summary which details how their individual contributions furthered the group's effort in addition to a reflection of learning. Students will use the claim-evidence-reason framework to support their position.
4	3. Throughout the revision/completion process, the role of the teacher is facilitative. The teacher will actively observe groups, help students with group dynamic issues if needed, ask probing questions and/or direct towards further resources.	***1. Summative Assessment:*** Students will take summative assessment. ***2. PBL Casework:*** Students refine their presentation based on feedback and self-assessment information. Student groups will continue to refine their presentation. 4. ***Closure/Group-assessment:*** Groups will meet for group self-assessment and score their presentation on final rubric. Each score must include claim-evidence-reason framework.
5	***1. Presentation/Participation:*** Final presentation cycle for each group to include: a) Group solution presentation, b) Open forum clarifying questions , c) Open forum probing questions, d) Community member feedback ***3. Summarize/Debrief:*** After all presentations are finished, teacher will facilitate a debrief of the problem.	2. Groups will present. While listening to the presentation, teachers, students, and community members will write down key points and questions. All participants will have the opportunity to ask follow up questions after each presentation. ***4. Closure:*** After group debrief, students will incorporate synthesis of learning experience into individual self-assessment

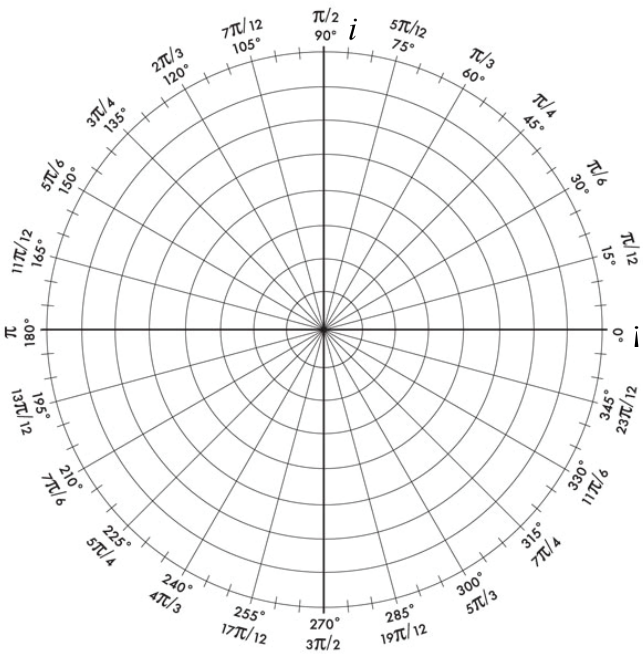
Summative Assessment Items: *Attach here.* For each summative assessment item, include an answer key or scoring rubric describing how points are to be awarded.

Type I:

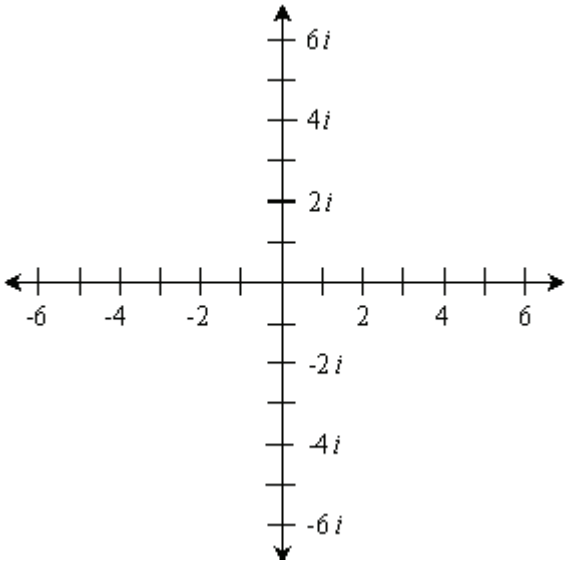
1. Plot and label each polar coordinate on the complex plane.

A. $z = 3(\cos 60^{\circ} + i \sin 60^{\circ})$

B. $z = -2(\cos 45^{\circ} + i \sin 45^{\circ})$



2. Plot the points corresponding to $3-2i$ and $1+4i$. Add these complex numbers and plot the result. How is this point related to the two others?

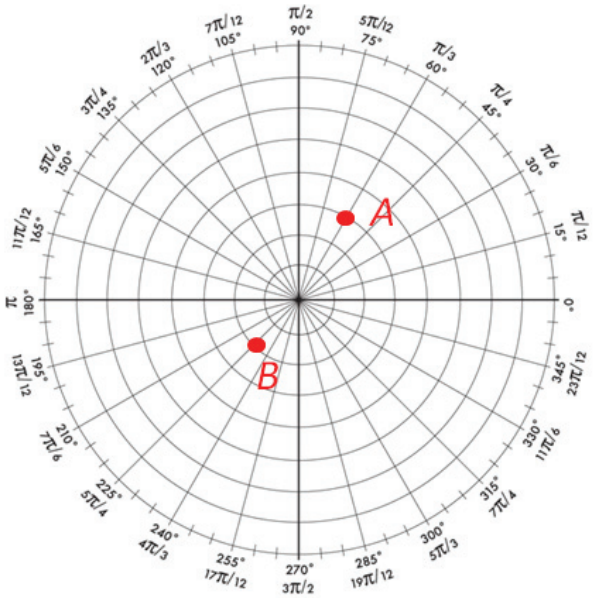


SOLUTIONS

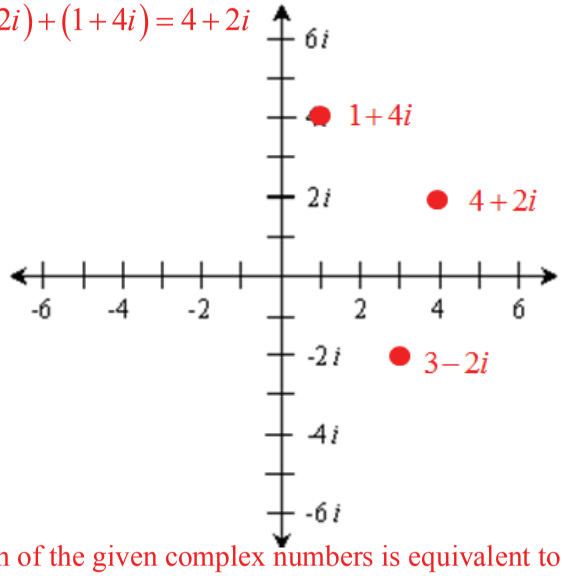
Scoring: This item represents a level 2 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can graph complex number on the complex plane in both rectangular and polar form.

1.



2. $(3-2i)+(1+4i)=4+2i$

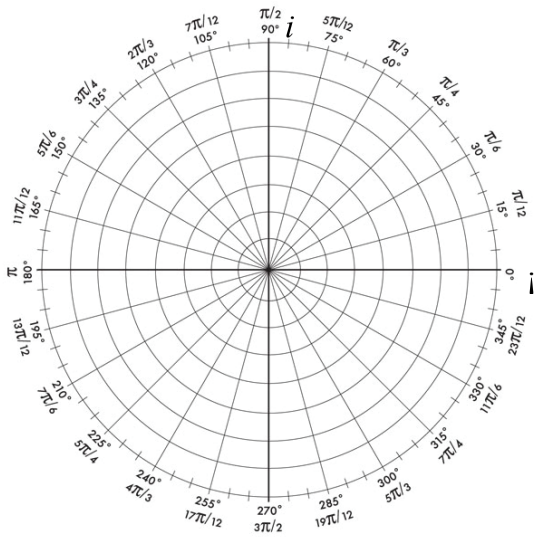


The sum of the given complex numbers is equivalent to the point one would arrive at if starting from the origin and moving on the real and imaginary axis as prescribe one after another. OR Geometrically, addition of two complex numbers can be done geometrically by constructing a parallelogram. The sum is the endpoint of the diagonal.

Type II:

1. If $(r,\theta)=(4,300^{\circ})$, then $(x,y)=$

2. Express the number $z=\sqrt{3}-i$ in polar form. Plot this number on the complex plane.



3. Given the complex number in polar form $z=r(\cos\theta+i\sin\theta)$, what is the polar form of $-z$? Justify your answer with both verbal and algebraic arguments.

SOLUTIONS

1. $(2,-2\sqrt{3})$

2. $2(\cos 30-i\sin 30)$

3. Algebraic:

$$z=r(\cos\theta+i\sin\theta)$$
$$-1\bullet z=-1\bullet r(\cos\theta+i\sin\theta)$$
$$-z=-r(\cos\theta+i\sin\theta)$$
$$-z=r(-\cos\theta-i\sin\theta)$$

Scoring: This item represents a level 3 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can convert between rectangular form, $z=x+yi$, and polar form, $z=r(\cos\theta+i\sin\theta)$, of a complex number.

Student can justify rectangular and polar forms of a complex number as representing the same number.

Verbal: A negative z would reflect the coordinate in the opposite direction on the *x*-axis as well as on the *y*-axis. Thus moving the coordinate from the first to the third quadrant.

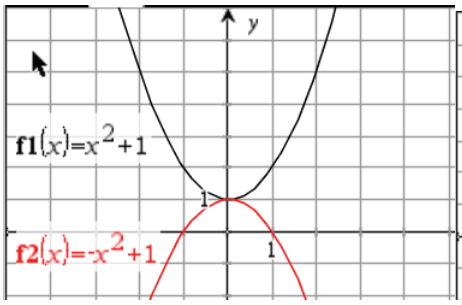
Type III:

1. While researching more information about how to graphically represent the complex roots of an equation, Maria found the following step-by-step directions.

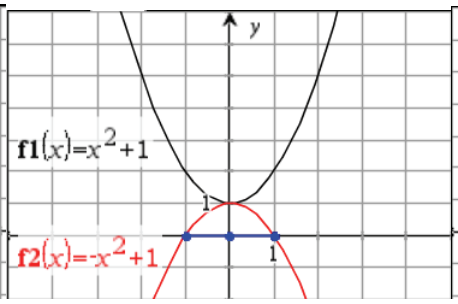
If the parabola does not intersect the x-axis, reflect it vertically over its vertex (Step 1). Find the x-intercepts of this reflection (Step 2). Rotate these intercepts 90 degrees about their midpoint (Step 3). The coordinates of these rotated points, written as complex numbers of the form a + bi and a – bi using their x-coordinate as a and their y-coordinate as b, will be the desired roots.

Maria shows this process with the equation $x^2+1=0$.

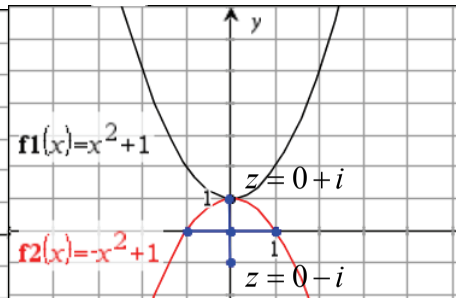
Step 1:



Step 2:



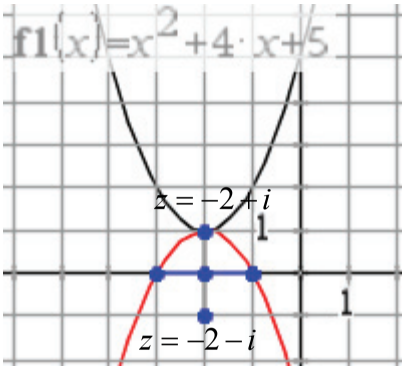
Step 3:



You are still skeptical about if this process will graphically represent the complex solutions to any equation. Follow the process for the equation $x^2+4x+5=0$, then solve the equation algebraically. Do both methods yield the same result?

SOLUTIONS

Graphically



Scoring: This item represents a level 4 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can verify a general process to graphically determine the imaginary roots of a given equation.

Algebraically

$a=1 \quad b=4 \quad c=5$

$$z=\frac{-4\pm\sqrt{16-20}}{2}$$

$$z=\frac{-4\pm -2i}{2}$$

$$z=-2\pm i$$