Curriculum Sample Template

Grade Level	9th	**Content Area**	Math
Course Title (grades 9-12 Only)	Integrated Math I		
Expected Prior Knowledge *The knowledge/skills mastered earlier in the year foundational to the mastery of the required Standard in the current sample.*	The repertoire of functions that is acquired during high school allows students to create more complex equations, including equations arising from linear and quadratic functions, and simple rational and exponential functions (A-CED). Students are no longer limited largely to linear equations in modeling relationships between quantities with equations in two variables (A-CED.2). Students can work with inequalities and systems of equations (A-CED.3). As well, students can graph linear equations, graph systems of linear equations, simplify inequalities to represent them in a format that is easy to graph.		
Alignment to Program of Instruction *Briefly outline methods of instruction found in this sequence of lessons that align to the Program of Instruction described in A.3.*	This sample also demonstrates the three distinctive features of SySTEM Phoenix's Program of instruction: 1) Integration: Math and Science standards are taught in a mutually reinforcing manner 2) Problem-Based Learning: The standard is introduced to students within the context of a real-world problem that they continually hook their new knowledge into in order to create a viable solution or make an evaluative judgment 3) All final products are authentic documents that students will present to an audience of stakeholders.		
Standard Number and Description *List the Number* and the full description for the Required Standard or Science Performance Objective listed in section A.6.*	A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		
Materials/Resources Needed	***Computers, Colored pencils*** Gizmo "Linear Inequalities in Two Variables – Activity A" www.explorelearning.com Gizmo "Linear Inequalities in Two Variables – Activity B" www.explorelearning.com "Linear Inequalities Activity" http://www.shodor.org/interactivate/activities/inequality/ Business.Fullerton.edu Linear Programming Presentations and Step-by-Steps: "Linear Programming Theory And Solver Output/Sensitivity Report", "Chapter 2: Linear Integer Programming Models", "Linear Programming: Optimal Solutions and Models Without Unique Optimal Solutions"		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Standard (e.g., 6.EE.7). For HS Math, use Conceptual Category-Domain, Standard (e.g., A-REI.6). For Science, use Grade, Strand, Concept, PO (e.g., 4.1.3.1).*

Lesson (add as needed)	**Instructional Strategies**	**Student Activities**
1	***1. Essential Question:*** How can systems be analyzed for maximums, minimums, and intersections? ***2. Launch/Introduction:*** **The Case:** You are a restaurant owner trying to establish yourself in the restaurant business. Your restaurant just opened, and you are aware that the first impression of a restaurant can make it or break it. You are asked to potentially host an executive dinner for up to fifty people. Your job is to properly plan for all aspects of the event, so that you can secure this event for your restaurant, and begin to build your restaurant's reputation. Your task is to plan an executive dinner at your restaurant. The challenge is to include the necessary mathematical and design concepts that will make the night a success, including planning the menu, arranging the tables, designing a program for the event, and calculating the cost. Your audience is a group of executives that are employed by the company wanting you to have the event. These individuals will ultimately decide where the event will take place, and it is your job to secure it at your restaurant. ***4. Vocabulary:*** Student Glossary Graphic Organizer of Tier 2 academic) and Tier 3 (content) vocabulary. "Term, Definition, Example." Inequality, solution, half-plane, solution region ***6. Summarize/Debrief:*** After students have explored the gizmos, the teacher will prompt a whole class discussion. Students will *look for and express regularity in repeated reasoning* in order to generalize rules about the graphic depiction of inequalities, using their observations as the evidence for their rationale. For example, a linear equality with the "less than" or "greater than" symbol has a broken boundary line because the points on the line are not part of the solution set. As a facilitator, the teacher will probe students at the whole class level distinguish the difference between graphical representations of equations and inequalities. In addition to the learning targets addressed in step 5, the teacher will ensure the following learning targets are discussed: *1) students recognize that both a line and a half-plane represent infinite solutions.*	***3.*** Students will be organized into collaborative working groups of four, which will stay consistent throughout the entire case. One student will have the role of group *facilitator* and a second student will have the role of *scribe. S*tudents will examine the problem and fill out first impressions using a graphic organizer to record answers to the following prompts: 1.What is the problem?, 2.What do we know about the problem?, 3. What information or supplies do we need to solve the problem?, 4.What should we do to solve the problem? When finished, all groups will share out their initial responses in a whole class format. ***5. Investigate/Explore:*** Students will use gizmos from elearning.com to interactively explore linear inequalities in two variables using "Activity A and B." As a facilitator, the teacher will ensure students observe the following: *1) students recognize that all points on a half-plane are solutions to a linear inequality and 2) students can determine whether the boundary line should be included as part of the solution set .* ***7. Formative Assessment:*** Three problems require students to graph a line, or boundary line, and share the appropriate region for a two variable linear inequality. Students will use colored pencils to represent the region of solutions. ***8. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct an initial version of the explanation for the problem and identify the most important concepts and their relationships. The group may delegate individual group members to gather specific information for the group.

2	***1. Introduction:*** In business planning, inequalities can be used to represent constraints, or factors that restrict a business in its development. For example, time is a constraint. There are only 24 hours in a day and most employees only work 8 hours each day. ***3. Modeling:*** As a whole group, teacher will lead students to *model with mathematics* by translating the ice cream cart constraints into linear inequalities. Since students have already encountered graphing systems of linear equations, the teacher will prompt students to apply this prior knowledge to model and graph a system of linear inequalities and shade the appropriate overlapping region for a system of linear inequalities. "An salesman must spend less than $50 each day. He can buy ice cream A for $1 and B for $2 each." Once students share their various attempts, the teacher will ensure the following learning target is formalized: *students graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for non-inclusive inequalities.*	***2.*** Students consider the prompt: "What are the constraints of an ice cream cart salesman's business?" In groups, students will brainstorm and list these constraints. (i.e. # of ice cream bars that can fit in the cart, time ice cream can stay in cart before it melts). ***4. Formative Assessment:*** Given two constraints related to the ice cream cart scenario, students will represent the constraints as linear inequalities and use colored pencils to graph the solutions as a half-plane (specific information to general representation). *Differentiation:* Inequality "word frames" to assist students with converting the text (the constraints) into math (inequalities). Extension question asks students to interpret the meaning of these solutions (general representation to specific information). ***5. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. The group facilitator organizes the group to construct an initial version of the explanation for the problem and identify the most important concepts and their relationships. The group may delegate individual group members to gather specific information for the group.
3	***1. Introduction:*** Mathematical optimization is the branch of computational science that seeks to answer the question `What is best?' for problems in which the quality of any answer can be expressed as a numerical value. Such problems arise in business, physical, chemical and biological sciences, engineering, architecture, economics, and management. Today, you will model two constraints with linear inequalities to represent all possible viable models within those constraints and learn to determine the optimal solution. ***3.*** The teacher will support the use of technology to model examples of intersections of inequalities ***4. Summarize/Debrief:*** In a whole group, students will present their system of inequalities and rationale. *Students will critique the reasoning of others.* ***5. Modeling:***. Facilitator will ensure the following learning targets necessary for linear programming are formalized: *1) students graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding half-planes, 2) students create an optimization function, 3) students test all extreme points within the system to determine the optimal solution.*	*2.* ***Investigate/Explore:*** Students will *use appropriate tools strategically* (graphing software, etc.) to resolve the following scenario. "Sarah makes bracelets and necklaces to sell at a craft store. Each bracelet makes a profit of $7, takes 1 hour to assemble, and costs $2 for materials. Each necklace makes a profit of $12, takes 2 hours to assemble, and costs $3 for materials. Sara has 48 hours available to assemble bracelets and necklaces. If she has $78 available to pay for materials, how many bracelets and necklaces should she make to maximize her profit?" As a facilitator, the teacher will prompt students to categorize the givens in the situation and represent their relationship as inequalities. ***6. Formative Assessment:*** Given three linear inequalities, representing cost and time constraints, students graphically represent all possible solutions and determine the optimal solution. ***7. PBL Case Work:*** In collaborative working groups, students will reflect on how their new knowledge relates to the case. *Differentiation:* Students are offered self-paced learning materials from Fullerton University on more complex Linear Programming Models.

<table>
<tr>
<td>4</td>
<td>1. Introduction: Businesses juggle multiple constraints simultaneously. New business owners who do not understand these constraints risk making costly decisions or losing out on potential profit.

4. Summarize/Debrief: As a whole group, students will offer their approaches for group discussion. As a facilitator, the teacher will seek groups with different approaches and/or answers to reconcile their methods. Students will critique the reasoning of others. Facilitator will ensure the following learning target is formalized: 1) students can design an optimization model for a given business, 2) students can graph the constraints as a system of linear inequalities, 3) students can utilize linear programming methods to determine the solution that optimizes the business' profit.

5. Summarize/Debrief: Using a "Jigsaw" protocol, students will work with students outside of their collaborative working groups. Each member will present his/her optimization model to the group and receive feedback. The teacher as facilitator will ensure the following learning target is consistently demonstrated: student can represent and graph constraints as a system of linear inequalities, 2) student can explain the meaning of the intersection of the shaded regions in a system of linear inequalities</td>
<td>2. PBL Case Work: In collaborative working groups, students will brainstorm additional constraints on their executive dinner and craft "need to know" information into well formulated questions. Groups will email the list, questions, and preliminary work to a local restaurant owner for review.

3. Students will use appropriate tools strategically (graphing software, etc.) to resolve the following scenario. "You are making fruit baskets with oranges, bananas, and apples. The table below gives the amount of fruit required for the two arrangements. Each day you have 240 oranges, 270 bananas, and 320 apples. Arrangement A earns a profit of $10 per basket and Arrangement B earns $8 per basket. How many of each fruit basket should you make per day to maximize your profit?"
<table>
<tr><th>Fruit</th><th>Arrangement A</th><th>Arrangement B</th></tr>
<tr><td>Oranges</td><td>4</td><td>6</td></tr>
<tr><td>Bananas</td><td>6</td><td>6</td></tr>
<tr><td>Apples</td><td>8</td><td>6</td></tr>
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As a facilitator, the teacher will prompt students to categorize the givens in the situation and represent their relationship as inequalities.

5. Independently, students will design an optimization model for the PBL case, write inequalities that represent the identified constraints, graph the solution set as the intersection of the half-planes, and test extreme points to determine the optimal solution. Students will make sense of problems and persevere in solving them.</td>
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<td>5</td>
<td>2. Introduction: Teacher will review the peer review schedule and product expectations for the PBL case. Teacher will review rubric and deadlines for self-assessment.

3. Group Conferencing/Feedback: As students work on their presentation, will dialogue with groups about their process, findings, and solution. The teacher will actively observe groups, help students with group dynamic issues if needed, and ask probing questions.

4. Direct Instruction/Workshop: Teacher will workshop the IMRAD (introduction, methodology, results, analysis, and discussion)</td>
<td>1. Formative Assessment: A refinery produces both gasoline and fuel oil. It sells gasoline for $1 per gallon and fuel oil for $0.90 per gallon. The refinery can produce at most 600,000 gallons a day, but must produce at least 2 gallons of fuel oil for every gallon of gasoline. At least 150,000 gallons of fuel oil must be produced each day to meet current demands. How much of each type of fuel should be produced to maximize daily profits?

2. PBL Case: Student groups will create a presentation to share their solution to the PBL Case. The presentation will be structured in the IMRAD (introduction, methodology, results, analysis, and discussion)</td>
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	structure to groups and/or members who choose to attend.	format. Students will review the feedback from local business owners. ***5. Closure/Self-Assessment:*** Individually, each student will self-assess their learning through a written summary which details how their individual contributions furthered the group's effort in addition to a reflection of learning. Students will use the claim-evidence-reason framework to support their position.
6	3. Throughout the revision/completion process, the role of the teacher is facilitative. The teacher will actively observe groups, help students with group dynamic issues if needed, ask probing questions and/or direct towards further resources.	***1. Summative Assessment:*** Students will take summative assessment. ***2. PBL Case Work:*** Student groups will continue to refine their presentation. ***4. Closure/Group-assessment****:* Groups will meet for group self-assessment and score their presentation on final rubric. Each score must include claim-evidence-reason framework.
7	***1. Presentation/Participation:*** Final presentation cycle for each group to include: a) Group solution presentation, b) Open forum clarifying questions , c) Open forum probing questions, d) Community member feedback 3. Summarize/Debrief: After all presentations are finished, teacher will facilitate a debrief of the problem.	2. As groups present, all audience members, teachers, students, and community members will write down key points and questions. All participants will have the opportunity to ask follow up questions after each presentation. ***4. Closure:*** After group debrief, students will incorporate synthesis of learning experience into individual self-assessment.

Summative Assessment Items: *Attach here.* For each summative assessment item, include an answer key or scoring rubric describing how points are to be awarded.

Type I:

1. Decide whether the boundary line should be included for each inequality below. Justify your answer

A. $3x - 4y \leq 7$

_____

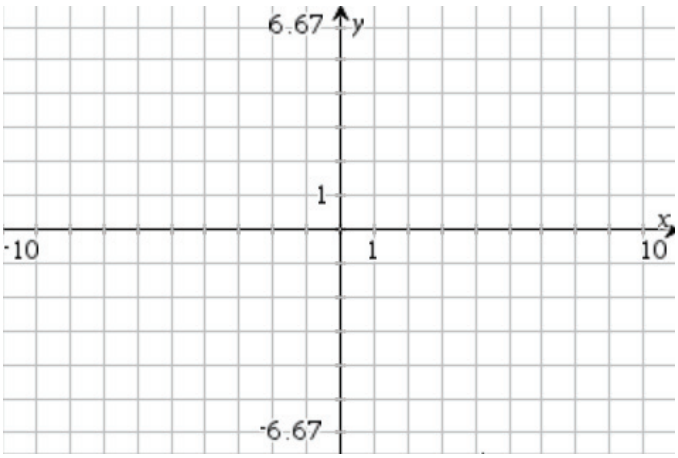
B. $y > -2x + 6$

_____

C. $-9x + 4y \geq 1$

_____

2. Graph the solution to $y < 2x + 3$.



3. How do we use a graph to represent the solutions to a linear inequality? Why do we use a graph instead of listing the solutions?

SOLUTIONS

Scoring: This item represents a level 2 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can identify the solutions as a region of the plane.

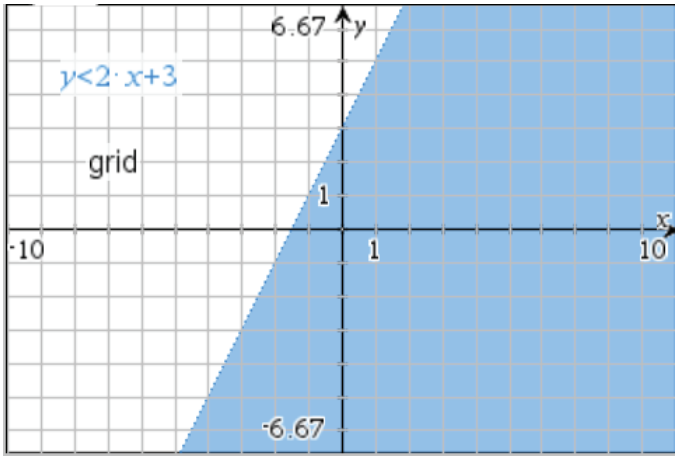
Student can determine whether the boundary line should be included as part of the solution set

.

Student can graph the solution to linear inequalities in two variables.

1.
A. Included. The "less than or equal" inequality includes the boundary.
B. Not included. The "greater than" inequality does not include the boundary line.
C. Included. The "less than or equal" inequality includes the boundary.

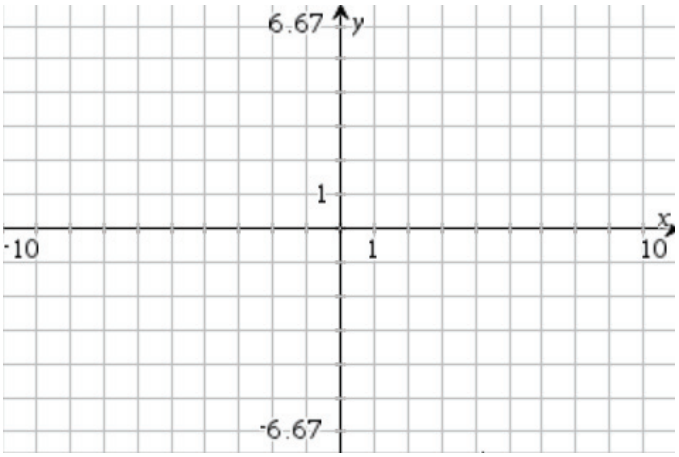
2. Line is dashed



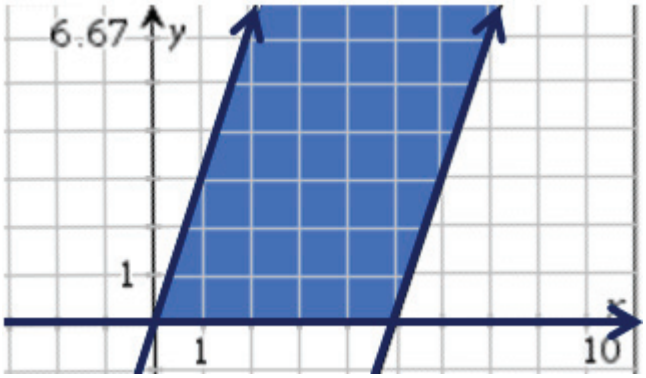
3. While the infinite solutions to an equation can be represented as a line, the infinite solutions to an inequality include an entire half-plane of the graph. The solutions are easily represented on a graph.

Type II:

1. Graph the solution set of $x + 2y > 12$ and $3x - y > 9$.



2. Create a context what represents the shaded area. Write the system of inequalities that models the meaning of the context. Describe the connection between the context, inequalities, and graph.



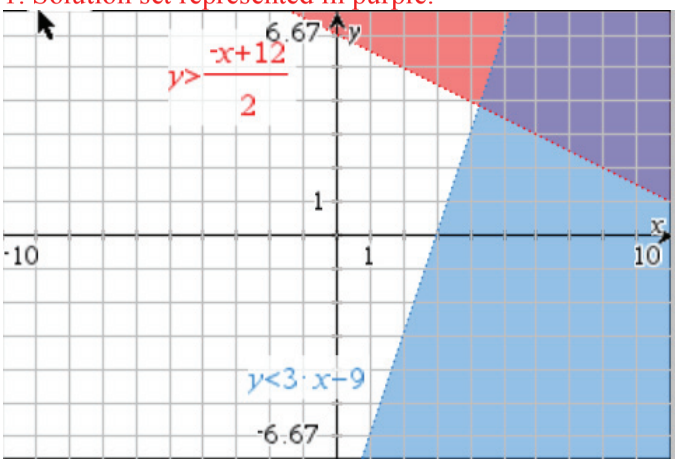
SOLUTION

Scoring: This item represents a level 3 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can graph the solutions to systems of linear inequalities in two variables.

Student can identify the solutions as a region of the plane.

1. Solution set represented in purple.



2. Student answers may vary. For example, bicyclists are racing cross country. The first cyclist starts at time zero and continues on at a pace of at least 3 miles per hour. The second cyclist preferred to take along 15 hour nap before the race and as a result cycles at a pace of greater than 3 miles per hour.

$y \geq 0$

$y \leq 3x$

$y \geq 3x - 15$

Type III:

1. Given:

$$x + y \le 6$$
$$x - y \le 6$$
$$x \ge 0$$
$$y \ge 0$$

What point maximizes the objective function $P(x, y) = 3x + 4y$? Explain your process.

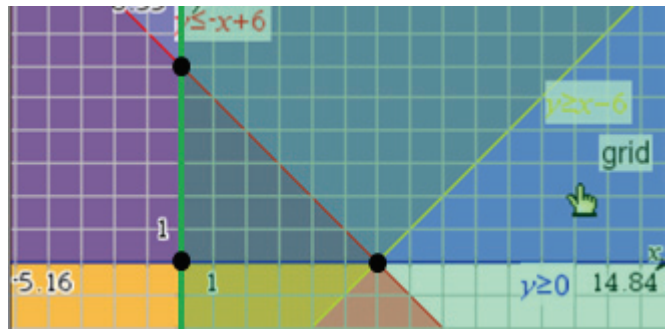
2. A company makes 2 types of chairs: a basic chair and a deluxe chair. In one day, the company can make at most 150 chairs total. The company can make at most 100 basic chairs and at most 75 deluxe chairs. The company makes \$12 profit for each basic chair and \$8 profit for each deluxe chair. Let x represent the number of basic chairs made in one day, and let y represent the number of deluxe chairs made in one day. Use linear programming to determine the maximum possible profit for one day. Show the steps for the linear programming method as requested.

A. Write the system of inequalities that represents this situation.
B. Graph the feasible region for this system of inequalities. Explain how you found your answer.
C. Identify the vertices of the feasible region.
D. Write the objective function.
E. What is the maximum possible profit for one day? Show your work, and explain how you found your answer.
F. How many of each type of chair are made to the maximum possible profit for one day?

SOLUTION

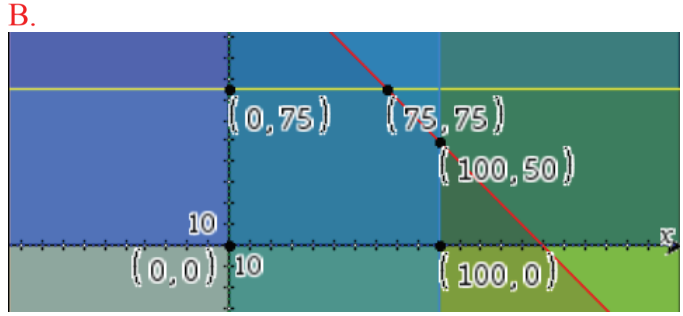
Scoring: This item represents a level 4 mastery of the standard. To achieve this level, students must answer 80% or more of the questions correctly. The student must demonstrate:

Student can create an optimization model for a business. Graph the constraints as a system of inequalities, and utilize linear programming methods to determine the situation that optimizes the business'



1. After graphing each inequality, identify the three extreme points. Of the three points of intersection (0,0), (6,0), and (0,6), one will maximize the objective function. Through substitution, the point (0, 6) maximizes the function.

2.
A. $x + y \le 150,\ x \le 100,\ y \le 75,\ x > 0,\ y > 0$
B.



C. $(0,0),\ (0,75),\ (75,75),\ (100,50),\ (100,0)$
D. $P(x, y) = 12x + 8y$
E. $P(0,0) = 0,\ P(0,75) = 600,\ P(75,75) = 1500,$
$P(100,50) = 1600,\ P(100,0) = 1200$
F. To maximize possible profit, 100 basic chairs and 50 deluxe chairs should be made each day.