Grade Level	Content Area	Course Title *(9-12 Samples Only)*	Unit Title
9th -12th Grade	Mathematics	Discrete Math: Vertex-Edge Graphs	Hamilton Circuits & Hamilton Paths

Length of Unit	Time of Year Taught	Expected Prior Knowledge
Five days	Any High School Semester	Mastery of the following standard(s) and PO(s): MHS-S2C4-01 and MCWR-S2C4-01. Units within Discrete Math: Intro to Vertex-Edge Graphs, Graphs and Systems, Paths and Circuits

Unit Description

Discrete Math is delivered as an independent, self-paced, computer-based course. Students use ANYWHERE LEARNING SYSTEM (A+LS™) for lesson instruction and activities. This course was authored by R.O.S.E.® to specifically address Arizona's 2008 Mathematics Standard using the ANYWHERE LEARNING SYSTEM . Although this course and unit is intended to be taken as an independent, self-paced program it has teacher involvement built into the lesson sequence. This allows the math instructor to check student understanding and perform formative assessments to ensure the student is able to master the standards on the summative assessment at the end of the unit.

The following topics are covered in this unit:

The focus of this unit is on Hamilton circuits and paths: their definition, their rules and structure, and their importance in the everyday world in application problems and specific algorithms used to find the least-costly and/or shortest route in a vertex-edge graph.

After completing this unit, the student will be able to:

- apply the vertex-edge graphing methodology to model and solve problems
- analyze a graph using the Brute Force, Nearest Neighbor and Shortest/Cheapest Link algorithms
- choose an algorithm for a particular type of problem and explain rationale
- solve a Traveling Salesperson Problem using the Nearest Neighbor and Shortest/Cheapest Link algorithms

Strand(s), Concept(s), and PO(s) *List and define the required Strand(s), Concepts(s), and PO(s) for each lesson included in this sample.*	MCWR-S2C4-02 Strand 2: Data Analysis, Probability, and Discrete Mathematics Concept 4: Vertex-Edge Graphs Understand and apply vertex-edge graphs. PO 2. Understand, analyze, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.
Summative Assessment *Describe a cumulative and comprehensive activity that allows students to demonstrate mastery of stated POs. A copy of the summative assessment must be included in the application package.*	The summative assessment is at the end of the unit. This assessment is fill-in-the-blank and measures the mastery of MCWR-S2C4-02. The student will need to demonstrate an understanding of the purpose, procedures and uses of Hamilton Graphs, determine if the example is a Hamilton Circuit or Path, and solve least costly and/or shortest route problems using the Nearest Neighbor, Shortest Link and Brute Force algorithms.

Assessment Scoring *Explain how each assessment is scored, to include points per question, total points possible, criteria, and grading scale. Describe rubric, if applicable, to include components being evaluated and the scoring criteria for each. A copy of the answer key and/or scoring rubric must be included in the application package.*	The unit test is 10 questions. Each question is worth 10 points. A passing score of 70% must be achieved on the unit test to be considered "successfully completed." A score of less than 70% on the unit test will result in the student having to retake the test. Grade Breakdown: 90 – 100% = A 80 – 89% = B 70 – 79% = C < 70% = no credit
Materials/Resources Needed *(Include all items for the entire unit.)*	Computer with Adobe Flash player, access to Microsoft Excel, and A+ NYWHERE LEARNING SYSTEM (A+LS™)–Discrete Math Lesson 8 Lab Activity #2 Directions Pencil/Paper for notes and as scratch paper Math Instructor for lesson activity clarification, further explanations, demonstration of problems, test proctoring.

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Lesson Instruction Is provided through the study portion of the unit lesson In the electronic course. A math teacher is readily available when the student needs further help.	Introduction (pgs1-9) (20 min) • Describe mastery and course expectations. • Introduce the mathematician who presented the Hamilton's Puzzle. • Provide an overview of Hamilton graphs that include Hamilton Circuits and Hamilton Paths. • Vocabulary is introduced and leveled definition is included. Guided Practice (30 min) • Describe and provide examples and uses for Hamilton circuits. • Describe and provide examples and uses for Hamilton paths. • Describe and provide examples and uses for Nearest Neighbor algorithm. • Describe and provide examples and uses for Cheapest Link algorithm. Activity #1 - Using Excel practice using the tools to create and draw graphs to be used in Activity #2.	Intro/Review (pgs 10-17) (15 min) • Compare and contrast Hamilton Graphs with Euler graphs. • Provide examples of Hamilton graphs. Is it a Hamilton circuit or Path or both? • Present examples of graphs exclusive for Hamilton circuits or paths. • Review types of uses for Hamilton graphs. • Introduce "best path" problems, how to identify the sequence that saves the most time and money. Guided Practice (20 min) • Present the Traveling Salesman(TSA) Problem. • Introduce the Brute Force Method as an algorithm to solve TSA, its pros and cons. Independent Practice (25 min) • Using the graphs created in Activity #1, graph Hamilton circuits and paths following directions in Activity #2 for questions 1 through 4.	Intro/Review (pgs 18-26) (15min) • Introduce the Nearest Neighbor algorithm that can also be used to solve the Traveling Salesman Problem. • Compare to the Brute Force method and identify pros and cons. Guided Practice (45 min) • Describe and present an example of the Nearest Neighbor algorithm • Demonstrate the algorithm step-by-step examine each option and determine the best route logically. • Using Excel and the pre-created graphs answer question 5 both (a) and (b) using the Nearest Neighbor algorithm in Lab activity #2. • Guided discussion of the results of the Brute Force and Nearest Neighbor algorithms and cost value.	Intro/Review (pgs 27-39) (5 min) • Introduce the Shortest Link algorithm that can also be used to solve the Traveling Salesman Problem. • Compare to other algorithms used previously. Guided Practice (30 min) • Describe and present an example of the Shortest Link algorithm. • Demonstrate the algorithm step-by-step, examine each option, and determine the best route logically. • Using Excel and the pre-created graphs answer question 5 both (c) and (d) using the Shortest Link algorithm for in lab activity #2. Check toward Mastery (25 min) • The student takes a practice test which provides feedback for each chosen answer.	Review (Lesson 8) (15 min) Review concepts from, Lesson 8, Hamilton Circuits & Hamilton Paths. Check Mastery (45 min) • Teacher provides test and proctors testing.

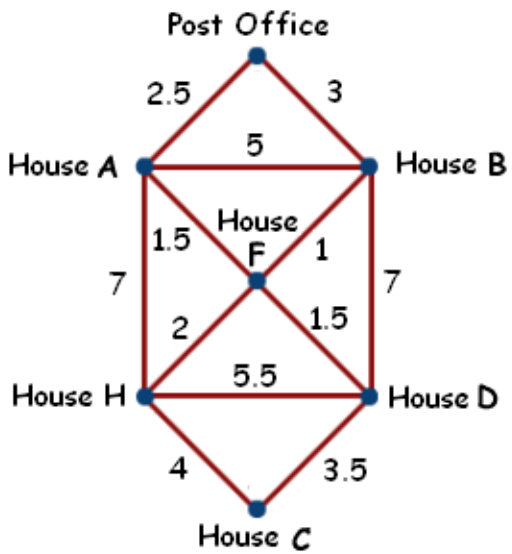
	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Student Activities	Introduction (20 min) • Student takes notes on course expectations, historical reference to Hamilton Puzzles, and Hamilton graphs and new vocabulary with inclusive definitions. Guided Practice (30 min) • Student studies and recreates examples and uses for Hamilton circuits, Hamilton paths, Nearest Neighbor algorithm, Cheapest Link algorithm. (20 min) (Lab Assignment) • Activity #1 - Student uses Excel to practice using the tools to create and draw the graphs to be used in Activity #2. (10 min) • Student requests teacher support as needed.	Intro/Review (15 min) • Student takes notes comparing Hamilton Graphs with Euler graphs. Creating examples and uses exclusive to Hamilton graphs, "best path" problems, and how to identify the most time and cost efficient sequences. Guided Practice (20 min) • Practice using the Brute force method to solve the Traveling Salesman (TSA) Problem. Independent Practice (25 min) • Student uses the graphs created in Activity #1 to graph Hamilton circuits and paths following directions in Activity #2 for questions 1 through 4. • Student requests teacher support as needed.	Intro/Review (15min) • Student takes notes on the Nearest Neighbor algorithm, its pros and cons and compares it to the use of the Brute Force method for solving the TSA problem. Guided Practice (45 min) • Student creates an example of the Nearest Neighbor algorithm, practices using it step-by-step to examine each option and determines the best route logically. (30 min) (Lab Assignment) • Student uses Excel and the pre-created graphs to answer question 5 (a) and (b) using the Nearest Neighbor algorithm in lab activity #2. (15 min) • Student reviews the discussion of the results of the Brute Force and Nearest Neighbor algorithms and cost value. • Student requests teacher support as needed.	Review (5 min) • Student takes notes on the Shortest Link algorithm, compares to other algorithms used. • Student reviews notes for graphing and algorithm review. Guided Practice (30 min) • Student creates an example of the Shortest Link algorithm, practices using it step-by-step to examine each option, and determines the best route logically. (20 min) (Lab Assignment) • Student uses Excel and the pre-created graphs to answer question 5 (c) and (d) using the Shortest Link algorithm in lab activity #2. (10 min) Check toward Mastery (25 min) • The student takes a practice test which provides feedback for each chosen answer. • Student requests teacher support as needed.	Review (15 min) • Student reviews notes for Lesson 8. • Student meets with teacher for clarification of understanding. Check Mastery (45 min) Meet with teacher to access final lesson test.

Discrete Math Hamilton Circuits & Hamilton Paths: Summative Assessment

MCWR-S2C4-02 Data Analysis, Probability, and Discrete Mathematics
Strand 2: This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs.
Concept 4: Vertex-Edge Graphs Understand and apply vertex-edge graphs. In Grades 11 and 12, students extend their understanding of networks to devise, analyze, and apply algorithms for solving problems related to circuits, shortest paths, minimum weight spanning trees, and adjacency matrices.
PO 2. Understand, analyze, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.

Unit Summative Assessment- All 10 questions in this unit summative assessment demonstrate mastery of MCWR-S2C4-02.

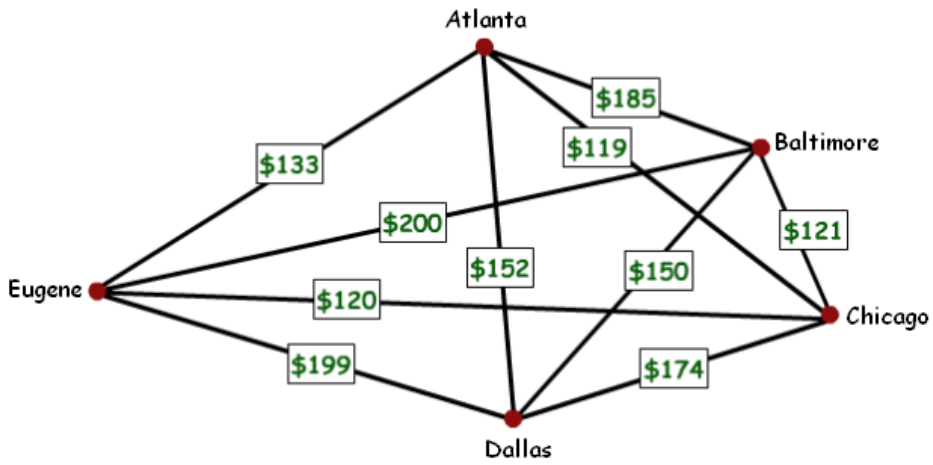
Fill in the Blank
The Post Master is planning how to most efficiently deliver mail in a newly built rural neighborhood shown on this map. Use the graph below to answer questions 1 & 2.



Question #1
Using the Nearest Neighbor Algorithm, list a Hamilton Circuit starting at the post office that the Post Master could set for the mail deliverers to use, determine the distance traveled. Refer to the numbers as distance in miles. _____

Question #2
Using the Cheapest Link Algorithm, list a Hamilton Circuit that the Post Master could set for the mail deliverers to use, determine the distance traveled. Refer to the numbers as distance in miles. _____

The following map displays the costs of traveling between cities. Use this map to answer questions 3 & 4.

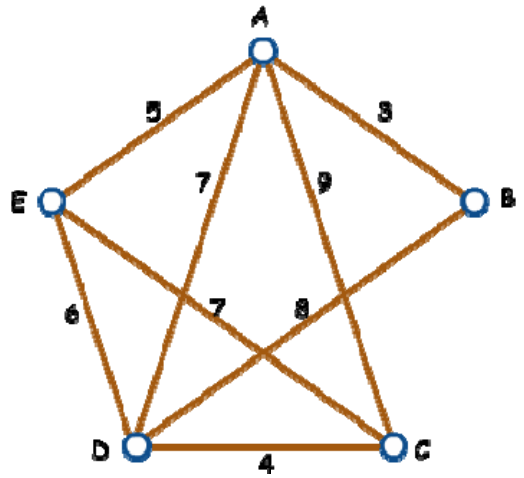


Imagine that a salesperson must perform jobs in all of the cities. This salesperson must design a travel route starting at Atlanta, visiting each city only once, and returning to Atlanta.

Question #3
What is the cost using the Nearest Neighbor Algorithm? _____

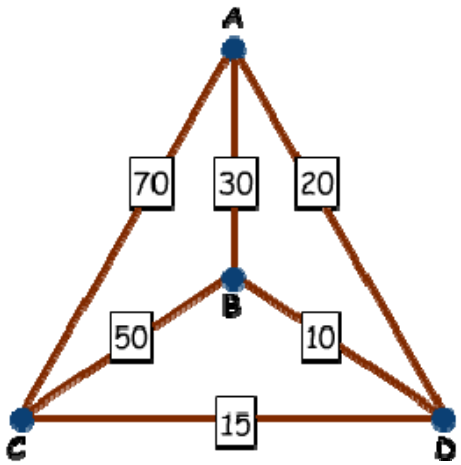
Question #4
What is the cost using the Cheapest Link Algorithm? _____

Question #5
List as many Hamilton circuits in this graph as you can find.



Question #6
Refer to the graph above, use the Nearest Neighbor algorithm beginning at point A. _____

Question #7
Refer to the graph above, use the Nearest Neighbor algorithm beginning at point E. _____



Question #8
Refer to the graph above, use the Cheapest Link algorithm to determine the cost. _____

Question #9
Refer to the graph above, use the Nearest Neighbor algorithm to determine the cost starting at C? _____

Question #10
Refer to the graph above, use the brute force method to determine the least-costly circuit value. _____

<u>Hamilton Circuits & Hamilton Paths, Answer Key</u>

1. Post Office, House A, House F, House H, House C, House D, House B, Post Office; 23.5 miles
(This list can be in reverse order as well but must start and end with the Post Office.)
2. Post Office, House B, House F, House D, House C, House H, House A, Post Office; 22.5 miles
(This list can be in reverse order and may start and end with the Post Office or any of the Houses as a reflection of the algorithm.)
3. $773 4. $741
5. There are twenty total possible Hamilton Circuits. The major sequences are ABDCE, ABDEC, ACEDB, and AECDB. Listed circuits may be in reverse order of what is listed here and must repeat the starting and ending letter.
6. 27 7. 27 8. 150 9. 125 10. 115