Grade Level	Content Area	Course Title *(9-12 Samples Only)*	Unit Title
9-12th Grade	Mathematics	Integrated Math Standards 4 Structure and Logic	Deductive and Inductive Reasoning

Length of Unit	Time of Year Taught	Expected Prior Knowledge
Six days	Varies	The following Standard(s) and PO(s) must be mastered: MHS-S1C1-02, MHS-S1C2-02, MHS-S2C2-01, MHSS2C2-01, MHS-S4C1-03, MHS-S4C1-05, 07, 08, MHS-S4C4-03.

Unit Description

Integrated Math Standards 4 is delivered as an independent, self-paced, hard copy course. Students use the Integrated Math Standards 4 workbook for lesson instruction and their student activities. Although this course and unit are intended to be taken as an independent, self-paced program, teacher involvement is built into the workbook sequence. This allows the math instructor to check student understanding and perform formative assessments to ensure the student is able to master the standards on the summative assessment at the end of the unit.

These following topics are covered in this unit:

The focus of unit 1 is two types of reasoning: deductive and inductive. The student is taken through the process of how we use deductive and inductive reasoning daily. The rules and forms for each will be explored. Each type will be broken down into smaller parts and examined for truth and validity. A variety of examples of each type of reasoning will be presented, described and broken down for truth and validity determination.

After completing this unit, the student will be able to:

- draw logical conclusions and make conjectures using inductive reasoning
- draw logical conclusions using deductive reasoning to analyze and prove valid or invalid conjectures with counterexamples
- identify and create counterexamples that refute invalid conjectures
- determine the truth value of inverse, converse, and contrapositive forms of statements using counterexamples
- list *if…then* statements in logical order
- draw a simple valid conclusion from a given *if…then* statement and a minor premise

Strand(s), Concept(s), and PO(s) *List and define the required Strand(s), Concepts(s), and PO(s) for each lesson included in this sample.*	MHS-S5C2-08-11 Strand 5: Structure and Logic Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications. PO 8 Use inductive reasoning to make conjectures, use deductive reasoning to analyze and prove a valid conjecture, and develop a counterexample to refute an invalid conjecture. PO9 State the inverse, converse, and contrapositive of a given statement and state the relationship between the truth value of these statements and the original statement. PO10 List related *if…then* statements in logical order. PO11 Draw a simple valid conclusion from a given *if…then* statement and a minor premise.

Summative Assessment *Describe a cumulative and comprehensive activity that allows students to demonstrate mastery of stated POs. A copy of the summative assessment must be included in the application package.*	The summative assessment is at the end of the unit. This assessment is multiple choice and measures the mastery of MHS-S5C2-PO's 08, 09, 10 and 11. The student will need to demonstrate an understanding of logic and reasoning as well as the application of logic and reasoning in math.
Assessment Scoring *Explain how each assessment is scored, to include points per question, total points possible, criteria, and grading scale. Describe rubric, if applicable, to include components being evaluated and the scoring criteria for each. A copy of the answer key and/or scoring rubric must be included in the application package.*	The unit test is 10 questions. Each question is worth 10 points. A passing score of 70% must be achieved on the unit test to be considered "successfully completed." A score of less than 70% on the unit test will result in the student having to retake the test. Grade Breakdown: 90 – 100% = A 80 – 89% = B 70 – 79% = C < 70% = no credit
Materials/Resources Needed *(Include all items for the entire unit.)*	Integrated Math Standards 04 workbook Paper/Pencil for note-taking and as scratch paper for making calculations Math Instructor for lesson activity clarification, further explanations, demonstration of problems, test proctoring.

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5

Lesson Instruction
 	
Is provided by the study portion of the unit lesson in this hard copy course. A math teacher is readily available whenever the student needs further help.
	Introduction (pgs. 5-14) (25 min)
	• Describe mastery and course expectations.
	• Provide an overview of Logic and Reasoning.
	• Math teacher proctors Pre-assessment.
	• Define Reasoning Vocabulary.
	• Describe two types of reasoning; deductive and inductive, with rules and forms for each.
	
	Guided Practice (35min)
	• Model the uses of inductive and deductive reasoning. Using, "think alouds" deliberately practice and walk through the rules and forms associated with deductive and inductive reasoning.
	• Compare/contrast deductive/inductive reasoning.
	• Demonstrate breaking reasoning statements into two parts: first part "a rule" "or observation" and second part "the conclusion".
		Introduction (pgs. 15 -17) (20 min)
		• Review deductive and inductive logic and activities completed in prior section.
		• Describe how the truth is determined based on the use of deductive or inductive forms of reasoning.
		• Euler and Venn diagrams provide a visual for how relationships are determined using inductive and deductive reasoning.
		• Explain how conclusions can be tested deductively and inductively.
		
		Guided Practice (40 min)
		• Model use of inclusion to determine deductive and inductive reasoning type and test for validity.
		• Provide examples of drawing conclusions using inclusion with Euler and Venn diagrams.
		• Demonstrate testing of a conclusion by using deductive reasoning and inductive reasoning.
			Intro/Review (pgs. 18-23) (25min)
			• Describe the difference between declarative and conditional statements.
			• Introduce and review conditional statement vocabulary.
			• Introduce and demonstrate the "if A, then B" structure as a form of deductive reasoning.
			• Describe how a statement can be written reversed and/or negated to show differing perspectives.
			
			Guided Practice (35 min)
			• Using inclusion, demonstrate written counterexamples of conditional statements; converse, inverse and contrapositive as forms of reversals of "if…then" relationships.
			• Practice determining antecedent and conclusions of conditional statements.
				Intro/Review (pgs. 23-27) (20min)
				• Describe and provide examples of valid and invalid conclusions.
				• Explain the relationship between the conclusion and the premise.
				• Explain how truth and validity are different.
				• Demonstrate with an Euler diagram how to prove the validity of a conclusion.
				• Show the transition of written statements to math statements.
				
				Independent Practice (40 min)
				• Questions and examples are provided for students to
				  • Write conditional statements.
				  • Determine a valid conclusion for each.
				  • Determine if the conclusion is valid or invalid.
				• Write 3 counterexamples of conditional statements and determine whether each one is valid or invalid.
				• Determine the counterexample type and if true or false.
					Intro/Review (pgs 27-35) (35 min)
					• Explain deductive forms of reasoning as a sequence used to solve math problems for algebra, geometry and discrete math. The focus is on why a solution is correct not completion of the problem.
					• Demonstrate written logic statements written in a sequential format.
					• Define and describe the form of syllogism that can be used with deductive or inductive reasoning.
					• Demonstrate forms of syllogism deductively and inductively.
					• Transfer knowledge of syllogisms using math content.
					
					Independent Practice (25 min)
					• Provide conditional statements in random order.
					• Provide math examples for student to demonstrate writing reversal statements with math problems.
					• Provide syllogism examples.

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Student Activities	Introduction (pgs 5-10) (25 min) • Student takes notes for course expectations, vocabulary, and content. (10 min.) • Student takes pre-assessment (15 min) • Student meets with teacher to clarify understanding. Guided Practice (35min) • Using rules, form, and reasoning samples; student determines whether the statement is an example of inductive or deductive reasoning (15 min) (pg. 11) • Following rules and form, student generates one example for deductive reasoning and one example for inductive reasoning. (10min) (pg. 12) • Student practices drawing conclusions deductively and inductively by reading sample statements and identifying the rule or making an observation and determining a conclusion. (10 min) (pgs. 13 & 14) • Student requests teacher support as needed.	Introduction (pgs. 15 -16) (20 min) • Student meets with teacher to clarify understanding of the application of deductive and inductive reasoning. (10 min) • Student takes notes using models of inclusion to determine relationships, arrive at conclusions, and determine whether it is a truthful statement. (10min) Guided Practice (40 min) • Student uses inclusion, either a Venn or Euler diagram, to break apart two different statements. (10 min)(pg.17) • Student determines if the statement is an example of deductive or inductive reasoning. (10 min) (pg.17) • Student describes how the conclusion is best tested for truth. (15 min) (pg.17) • Student states whether the conclusion is based on flawed reasoning. (5 min) (pg.17) • Student requests teacher support as needed.	Intro/Review (pgs. 18-22) (25min) • Student takes notes on declarative and conditional statements, and new vocabulary. (5 min) • Student reviews deductive reasoning notes. (5 min) • Student takes notes on "if A, then B" structure as a form of deductive reasoning and how a statement can be written reversed, and/or negated, to show differing perspectives using antecedent and conclusion. (15 min) Guided Practice (35 min) • Using inclusion, student practices writing reversals of conditional statements to examine different perspectives. Reversals include converse, inverse and contrapositive statements, and use different forms of "if A then B". (20 min) (Pgs. 20-21) • Student breaks down the conditional statement into the "if", antecedent, and the "then", condition, to examine the statement in a logical manner. (15 min) (pg. 22) • Student requests teacher support as needed.	Intro/Review (pgs. 23 24) (20min) • Student takes notes on valid and invalid conclusions, the relationship between the conclusion and the premise, the difference between truth and validity. • Draws Euler diagrams to demonstrate how to prove an invalid conclusion is invalid. Compares written statements to math statements. (10 min) Independent Practice (40 min) • Write two conditional statements, make a premise and draw valid conclusion. Using counterexamples prove the validity of each conclusion. (10 min) • Read examples of conditional statements determine a valid conclusion for each. (5 min) • Read conditional statements then determine if the conclusion is valid or invalid. (10 min) Read conditional statements, write 3 counterexamples of each, and determine whether each one is valid or invalid. (15 min)	Intro/Review (pgs 27-35) (35 min) • Review notes on forms of deductive reasoning. (5 min) • Take notes on sequencing of deductive statements. (5 min) • Review notes on deductive and inductive reasoning. (5min) • Take notes on deductive and inductive forms of syllogism.(5 min) • Take notes for using reasoning statements to determine why a math problem is correct. (10 min) • Student requests teacher support as needed. Independent Practice (25 min) • Sequence conditional statements in correct order. (5 min) • Student writes 3 counterexamples of math problems. (10 min) • Student determines if the syllogism examples are inductive or deductive and explains why. (10min)

	Lesson 6				
	MHS-S5C2-08 (R, M) MHS-S5C2-09 (R, M) MHS-S5C2-10 (R, M) MHS-S5C2-11 (R, M)				
Lesson Instruction	Review (Unit 1) (15 min) • Review concepts from Unit 1 of Integrated Math Standards 04. Check Mastery (45 min) • Teacher provides test and proctors testing.				
	Lesson 6				
Student Activities	Review (Unit 1) (15 min) • Student reviews notes and clarifies any remaining questions with a teacher for Unit 1 of Integrated Math Standards 04. Check Mastery (45 min) • Student completes Unit Test for Mastery. (see attachment)				

MAST 04 Units 1: Summative Assessment

MHS-S5C2-08-11 Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof
Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.
PO 08 Use inductive reasoning to make conjectures, use deductive reasoning to analyze and prove a valid conjecture, and develop a counterexample to refute an invalid conjecture.
PO-09 State the inverse, converse, and contrapositive of a given statement and state the relationship between the truth value of these statements and the original statement.
PO-10 List related *if…then* statements in logical order.
PO-11 Draw a simple valid conclusion from a given *if…then* statement and a minor premise.

1. Use inductive reasoning to make a conjecture based on this specific information: (MHS-S5C2-08)

Suppose John missed the school bus four Thursdays in a row.
Reasoning *inductively* you might conclude that _____.

A. Every Thursday John will miss the bus.
B. John misses the bus daily.
C. John doesn't like the Thursday bus driver.
D. There is a chance that John might miss the bus this next Thursday.

2. Use deductive reasoning to analyze and prove this valid conjecture: (MHS-S5C2-08)

The interior angles of an isosceles triangle add up to 180°.
Which of the following rules are necessary to support this conjecture?

I. Any two interior angles of a square add up to 180°.
II. All triangles have interior angle measures that add up to 180°.
III. An isosceles triangle has three interior angles.
IV. An isosceles triangle is a type of triangle.

A. I, II, IV
B. I, III, IV
C. I, III
D. II, IV
E. I, II, III, IV

3. Use deductive reasoning to analyze and prove a valid conjecture: (MHS-S5C2-08)

Given the following rules, which of the conclusions is valid?

All shapes with four 90° angles are rectangles. Squares have four 90° angles.

A. All rectangles are squares.
B. Some squares are rectangles.
C. Any shape with four 90° angles is a square.
D. A square is a rectangle.

4. Develop a counterexample to refute this invalid conjecture: (MHS-S5C2-08)

Which of the following is a counterexample that refutes this conjecture?
In other words, which of the following would be an appropriate counterexample for $x^3 \geq x^2$?

A. $x = 1$
B. $x = 1.5$
C. $x = 2$
D. $x = -2$

State the inverse, converse, and contrapositive of "*If a shape is a triangle, then it is a polygon"* for questions 5, 7 and 9; then state the truth value for questions 6, 8 and 10.

5. What is the converse of the statement? (MHS-S5C2-09)

A. If a shape is a polygon, then it is a triangle.
B. If a polygon is a shape, then it is a triangle.
C. If a polygon is not a shape, then it is not a triangle.
D. If a shape is a polygon, then it is not a triangle

6. Is the converse always, sometimes, or never true? If sometimes, or never true, provide a counterexample. (MHS-S5C2-09)

7. What is the inverse of the statement? (MHS-S5C2-09)

A. If a shape is not a triangle, then it is a polygon.
B. If a shape is a triangle, then it is not a polygon.
C. If a shape is not a triangle, then it is not a polygon.
D. If the polygon is not a shape, then it is a triangle.

8. Is the inverse always, sometimes, or never true? If sometimes, or never true, provide a counterexample. (MHS-S5C2-09)

9. What is the contrapositive of the statement? (MHS-S5C2-09)
   A. If a shape is not a polygon, then it is not a triangle.
   B. If a polygon is not a shape, then it is a triangle.
   C. If a shape is not a triangle, then it is not a polygon.
   D. If a polygon is not a shape, then it is not a triangle.

10. Is the contrapositive always, sometimes, or never true? If sometimes, or never true, provide a counterexample. (MHS-S5C2-09)

11. List these related *if…then* statements in logical order. (MHS-S5C2-10)
   i. If pink, then green
   ii. If red, then blue
   iii. If blue, then pink
   iv. If green, then purple
      A. I, II, IV, III
      B. IV, I, III, II
      C. II, III, I, IV
      D. I, II, III, IV

12. Draw a simple valid conclusion from this *if…then* statement and minor premise: (MHS-S5C2-11)
   If Jose practices his soccer skills every day, then he'll be chosen for the varsity team.
   Jose was chosen for the junior varsity team.
      A. Jose practiced his soccer skills every day.
      B. The varsity team practices Jose's soccer skills every day.
      C. Jose practiced his soccer skills every day and was chosen for the junior varsity team.
      D. Jose did not practice his soccer skills every day.

Answer Key
1. A safe answer would be (D). Because there is a *pattern* of missing only Thursdays, but *not a certainty*, we can only say that John *might* miss the bus next Thursday.
2. D 3. D 4. D
5. A
6. Answer: It is sometimes true. There are many types of polygons other than triangles: there are squares, octagons, dodecagons, etc.
7. C
8. Answer: It is sometimes true. If the shape is a circle it is not a polygon; but, If a shape is a square, an octagon, or other multi-sided shape, it is still a polygon.
9. A
10. Answer: It is always true. There are no counterexamples.
11. C 12. D