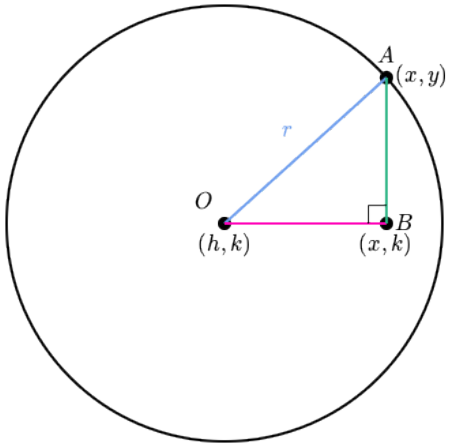
Curriculum Sample Template – 8 Pages Max. (12 pages for integrated ELA sample)

Grade Level	10	**Content Area**	Mathematics
Course Title (grades 9-12 Only)	Geometry		
Expected Prior Knowledge *List the knowledge/skills mastered earlier in the year that are foundational to the mastery of the Required Standard(s).*	Students will already be able to: • Understand and apply the Pythagorean Theorem (8.G.7)(8.G.8) • Prove that all circles are similar (G-C.A.1) • Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G-CO.A.1) • Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (A-SSE.B.3) b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. • Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (G-GPE.B.7)		
Alignment to Program of Instruction *Describe how the methods of instruction found in this sequence of lessons align to the Program of Instruction described in the application package.*	This lesson features Sheltered Instructional Observation Protocol (SIOP) techniques, including activating prior knowledge, pre-teaching vocabulary, strategic grouping, differentiation (during the guided practice phase of lessons,) and the use of clear content objectives (see the various publications from Echevarria, Voght, & Short on the SIOP model.) Scaffolding techniques include: the teacher giving students a fully worked out and correct model of the skills required as an exemplar, and giving students an opportunity to feel highly competent with one concept before introducing a more complicated additional task, circulating around the room to determine students who need additional help or a different way of explaining the concept or skill, and giving students opportunities to work in groups to give them the opportunity to interact over a common problem.		
Standard Number* and Description *The standard number and description of the Required Standard(s) being instructed and assessed to mastery in the curriculum sample.*	G-GPE.A.1 - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		
Materials/Resources Needed *List all items the teacher and students will need for the entire sequence of instruction (excluding common consumables)*	Compasses (including one for use on a white board/chalk board.) Student's Glossary Student's Math Journal Overhead or Document Camera		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Cluster, Standard (e.g., 6.EE.B.7). For HS Math, use Conceptual Category-Domain, Cluster, Standard (e.g., A-REI.C.6).*

Lesson (add as needed)	**Instructional Strategies -** *List and describe teacher actions, lesson by lesson, which will provide students with opportunities to engage the grade-level rigor defined by the Required Standard(s). Indicate alignment of instructional strategies to specific component(s) or content/skill(s) of the Required Standard/component.*	**Student Activities -** *Describe the activities, lesson by lesson, in which students will engage to allow them to master the grade-level rigor defined by the Required Standard(s). Indicate alignment of student activities to specific component(s) or content/skill(s) of the Required Standard/component.*
1	**1. Anticipatory Set –** Question on board: "Using complete sentences, answer the question: What is a circle?" Teacher gives several minutes to answer and then asks students randomly for their answers. **2. Learning Objective –** SWBAT draw a circle on graph paper using a compass and use the words "center" and "radius" to describe the parts of the circle. **3. Instructional Input –** Using the data from the Anticipatory Set, the teacher will instruct students to put the words "equidistant," "circle," "compass," and "radius" in their glossary. The teacher will then ask students to imagine what equidistant might mean by breaking it into parts and asking what the prefix "equi-" probably means, and then applying it to distance. The teacher will lead a discussion on the meaning of equidistant, making sure that all students have the definition of equidistant in their glossary as: "equal distances" and an example or picture. The teacher will then make sure that everyone has the definition of a circle as "the set of all points that are equidistant from a center point," and that they have an example or picture in their glossary. The teacher will then demonstrate using a meter stick on the board. After drawing several (10) points around the center point the image of a circle will become apparent. The teacher will then introduce the compass to the class, and show them how to use one to draw a circle, and asks the students to write the definition and a picture or example of a compass in their glossary. The teacher will then explain to students that the distance that all points on a circle are from the center is called the radius of the circle. The teacher will then instruct the students to put the definition and an example of radius in their glossary. **4. Modeling/Check for Understanding** – The teacher will then model using an overhead or document camera how to use a standard sized compass on graph paper to draw a circle **5. Guided Practice –** The teacher will then ask the students to draw a circle centered at the point (3,4) with radius 5. The teacher will circulate the room and assess whether students can do that, and help students who need it.	1. Students will use complete sentences to describe a circle and share when called upon. 2. Students will write the learning objective with the date in their notes. 3. Students will take notes in their math journal, ask questions to clarify, and add words to their Glossary. 4. Students will observe teacher use the compass, take notes, and ask questions to clarify. 5. Students will attempt to draw the specified circle on graph paper on their own with teacher assistance as necessary. 6. Students will draw seven more circles of their choice (within the parameters) and show the teacher as a formative assessment. **Alignment to content/skills of Required Standard:** This lesson will give students the foundational skills necessary to understand what a circle is and why it has the equation it does. By the end of this lesson students will be prepared to derive the equation of a circle using the Pythagorean Theorem.

	6. Independent Practice – The teacher will instruct students to draw seven additional circle using their compass with center that is not the origin, and radius between 2 and 7. The teacher will instruct students to keep these circles for the next lesson and show their work as an exit ticket.	
2	**1. Anticipatory Set –** Question on board: "Use the Pythagorean Theorem to calculate the length of the hypotenuse, $\overline{AC}$, for right triangle $\triangle ABC$ if $\overline{AB} = 18$ and $\overline{BC} = 24$. (The right triangle will also be drawn on the board.) Answer = 30 **2. Learning Objective –** SWBAT derive the equation of a circle using the Pythagorean Theorem by using the radius of a circle as the hypotenuse of a right triangle. **3. Instructional Input –** The teacher will demonstrate, using a generalized circle with center *(h, k)*, radius *r*, and an arbitrary point *(x, y)* that the horizontal distance from the center of the circle to the x-value of the point, B, is $\lvert x - h \rvert$, and that the vertical distance from the center to the y-value of the point, B, is $\lvert y - k \rvert$, and that because we can prove that $\overline{OB}$ is perpendicular to $\overline{AB}$, that a right triangle is formed with the points *A*, *B*, and *O*. The hypotenuse, $\overline{OA}$, is the radius of the circle.(Shown to the right.) The teacher then shows how it follows that we can use the Pythagorean Theorem to show that: $\lvert x - h \rvert^2 + \lvert y - k \rvert^2 = r^2$, which is the equation for any point r units from the center point *(h, k)*, or in other words, the equation of a circle centered at *(h, k)* with radius *r*. The teacher will also explain that because absolute value bars are grouping symbols also, and because everything squared is positive, the absolute value bars can be replaced with parentheses. So we write the equation of a circle in the form : $(x - h)^2 + (y - k)^2 = r^2$.  	1. Students will use the Pythagorean Theorem to find the hypotenuse of the circle and share when called upon. 2. Students will write the learning objective with the date in their notes. 3. Students will take notes in their math journal, ask questions to clarify, and add words to their Glossary. 4. Students will observe teacher model the problem and respond with next steps when called upon. 5. Students will attempt to derive the equation of the circle using the Pythagorean Theorem and ask questions if necessary. 6. Students will derive the equation for the seven other circles they drew and turn them into the teacher as a formative assessment. **Alignment to content/skills of Required Standard:** This lesson aligns with the standard by teaching students how to derive the equation of a circle with any radius and center by using the Pythagorean Theorem.

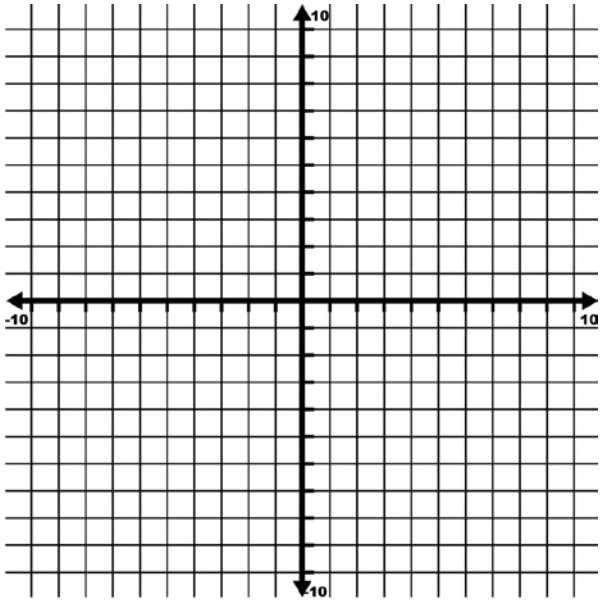
	4. Modeling/Check for Understanding – The teacher will then use an overhead or document camera to show how the process works with a specific circle and cold calls students to show her the next steps. **5. Guided Practice –** The teacher will have the students derive the equation for the first circle they drew in the previous lesson with center at the point (3,4) and radius 5 using the Pythagorean Theorem and will walk around the room checking student progress and answering questions. **6. Independent Practice –** The teacher will have the students derive the equations for the remaining seven circles that they drew in the previous lesson using the Pythagorean Theorem. Students will turn this work in to be graded as an assignment and as formative assessment.	
3	**1. Anticipatory Set –** Question on board: "Graph the circle $(x+4)^2 + (y-2)^2 = 16$" The teacher will call on one student to come up to the board to show the answer. The teacher will ask the students to assist the student as necessary. **2. Learning Objective –** SWBAT graph a circle given its factored equation and give the factored equation of a circle given its graph. (The point of today's lesson is to allow students to become very comfortable with the equation of a circle in anticipation of the next lesson.) **3. Instructional Input –** The teacher will instruct students to add "factored form of equation of circle" to their glossary, and give them the definition as: "The form of the equation of a circle which reveals the center and radius of the circle." Then the teacher will give them the equation form $(x-h)^2 + (y-k)^2 = r^2$ to put in the example/picture box. The teacher will then review how to graph a circle given its equation, and how to write the equation of a circle given its graph. The teacher will then explain to students how they can use their prior knowledge of the Midpoint Formula and Distance Formula to calculate the center of the circle and the radius of the circle if given other information about the circle, like the endpoints of its diameter.(For the purposes of this lesson, all radii with be positive integer squares.) **4. Independent Practice –** The teacher will give students 10 graphed circles (integer coordinates for center, positive integer square for radii) to practice writing the equation of the circle, 5 equations of a circle (integer coordinates for center, positive integer square for radii) to practice graphing a circle given an equation, and 5 problems that give them the endpoints of the diameter to practice integrating their skills.	1. Students will graph the circle and be prepared to show their answer if called or assist if necessary. 2. Students will write the date and learning objective in their math journal. 3. Students will take notes in their math journal, ask questions to clarify, and add words to their Glossary. 4. Students will give the equations of the circles and graph the circles whose equations or diameter endpoints are given. Students will turn this in to the teacher as a formative assessment grade. **Alignment to content/skills of Required Standard:** This lesson focuses on getting students comfortable with the factored form of the equation of a circle so they are well prepared to perform the skills required for the second part of the standard: complete the square to find the center and radius of a circle given by an equation.

4	**1. Anticipatory Set –** Question on board: "Review: Collaborate with your neighbors and make sure everyone at your table remembers how to to solve the quadratic equation by completing the square. $x^2 + 8x = 9$. The teacher will then ask several students to come up to the board and write their work and the answer (x= 1and -9). The teacher will ensure that the students remember how to complete the square. **2. Learning Objective –** SWBAT complete the square on the equation of a circle to write it in a factored form that reveals the center and radius of the circle. **3. Instructional Input –** The teacher will demonstrate how the procedure of completing the square is the same on the equation of a circle, the only difference is that they need to do it twice; once for x and again for y. The teacher will show how once the complete squares have been found for x and y the equation of the circle becomes exactly what they are used to. **4. Modeling/Check for Understanding** – The teacher will model how to complete the square on the equation of the circle $x^2 + y^2 + 2x + 10y + 10 = 0$ and ask students at random for next steps and answer students questions as they come up. **5. Guided Practice –** The teacher will have the table groups do another one on their own while circulating the room answering questions. **6. Independent Practice –** The teacher will give the students 10 equations of a circle in non-factored form with integer coordinates for center, positive integer square for radii to practice completing the square to reveal the center and radius of the circles, then graph them.	1. Students will work in groups to complete the square to solve the quadratic equation. Some students will go up to the board to work out their answer. Students will help out the students at the board and ask questions they have. 2. Students will write the learning objective with the date in their notes. 3. Students will take notes in their math journal, and ask questions to clarify. 4. Students will observe the teacher model the problem and respond with next steps when called upon. 5. Students will work in groups to complete the square on a non-factored equation of a circle and ask questions they have of the teacher and classmates. 6. Students will work independently to complete the squares of the given problems to reveal the center and radius of the circles, and turn in the work as a formative assessment grade. **Alignment to content/skills of Required Standard:** This lesson aligns with the standard by teaching students how to complete the square of a non-factored equation of a circle to find the center and radius of the circle.
S.A.	*Provide an opportunity for the student to complete the summative assessment items which is clearly separate from instruction and guided or independent practice, and in which the student is assessed independently. In the Student Activities column, provide a brief description of the summative assessment (S.A.) items that will allow students to demonstrate mastery of the rigor of the Required Standard(s)/components, and the context in which the items will be administered.*	An optional review period may be conducted before the end of unit test. Students will take an end of unit test immediately following the unit. The test will be free response and taken individually and take about 1 class period. Students will have the use of a scientific calculator: either their own, or one provided to them. Smart phones will not be allowed to be used as a calculator. It will a closed note test. The skills addressed in the standard will be assessed a second time (at least) in a Quarterly Common Assessment at the end of the quarter.

Summative Assessment Items and Scoring: *Below, provide at least three Summative Assessment Items that together accurately measure student mastery of the application of the content and/or skills as defined by the grade-level rigor in the Required Standard(s). For each assessment item, provide an answer key or scoring rubric which clearly describes how assessment items are to be scored and how points will be awarded, such that mastery of the application of the content and/or skills as defined by the grade-level rigor in the Required Standard is clearly demonstrated by an acceptable score.*

Summative Assessment Item 1 (clearly identify answer key/scoring rubric, and acceptable score):

Find the equation of the circle whose diameter has endpoints at (-7, -1) and (1, 5).
Show all steps and work. You may use the graph paper to the right.



KEY:

The center of the circle is the midpoint of the diameter. So, the x-coordinate of the center is: $\frac{-7+1}{2} = -3$, and the y-coordinate is $\frac{-1+5}{2} = 2$

The center of the circle is (-3, 2)……(1 point)

The radius of the circle is the distance from the center to a point on the circle. So, $\sqrt{(-7-(-3))^2 + (-1-2)^2} = 5$.

The radius of the circle is 5……(2 points)

The equation of the circle is $(x+3)^2 + (y-2)^2 = 5^2$……(1 point)

Scoring Guide:

The question is worth 4 Points.

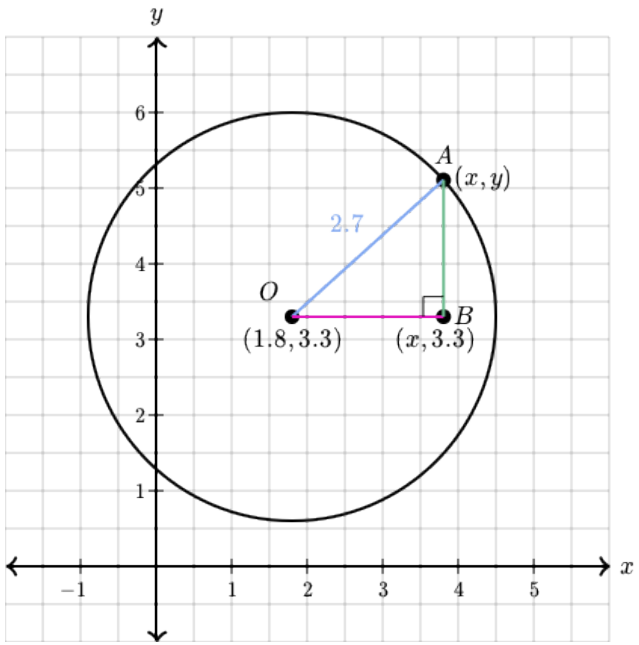
Partial credit may be given. Subtract 1 point per mistake in the equation or graph.

3 out of 4 indicates Mastery

Summative Assessment Item 2 (clearly identify answer key/scoring rubric, and acceptable score):

Use the Pythagorean Theorem to find the equation of the circle graphed to the right.

Show all steps and work.



KEY

1) $\overline{OB} = |x - 1.8|$, $\overline{AB} = |y - 3.3|$, and $\overline{OA} = 2.7$……(2 points)

2) ΔABO is a right triangle so $\overline{OB}^2 + \overline{AB}^2 = \overline{OA}^2$……(1 point)

3) So, $(x - 1.8)^2 + (y - 3.3)^2 = 2.7^2 = 7.29$, which is the equation of the circle…(1 point)

Scoring Guide:

The question is worth 4 Points.

Partial credit may be given. Subtract 1 point per mistake in the equation or graph.

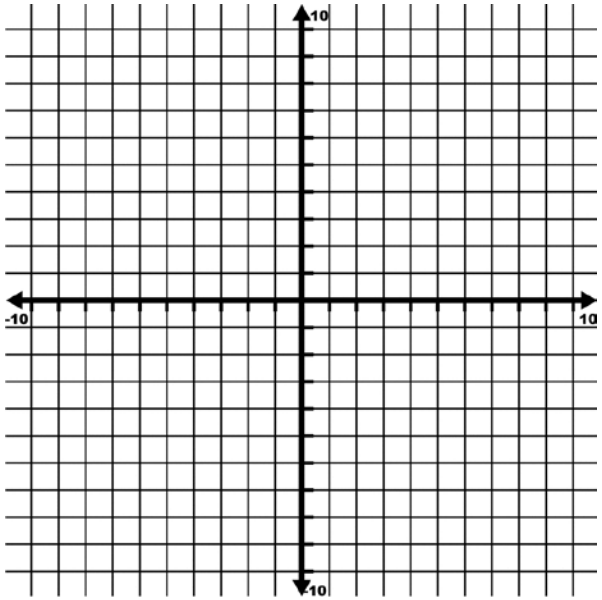
3 out of 4 indicates Mastery

Summative Assessment Item 3 (clearly identify answer key/scoring rubric, and acceptable score): (add as needed):

Write the following equation for a circle in factored form and graph it on the coordinates provided using a compass.

Show all work.

$x^2 + y^2 - 4x - 8y + 11 = 0$



KEY:

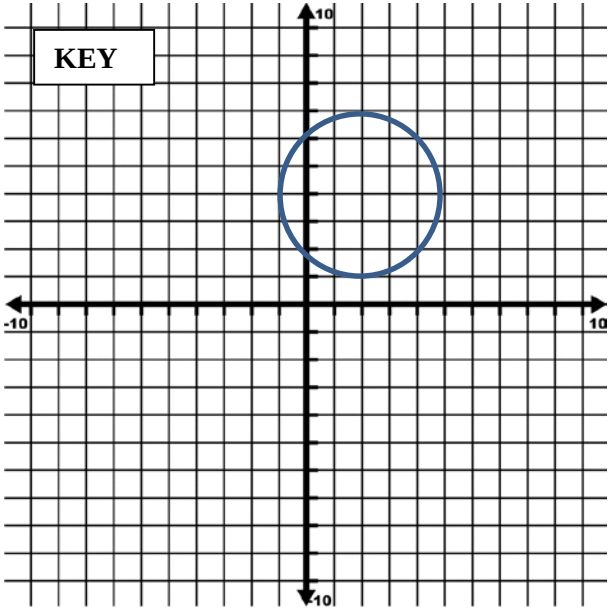
Students need to complete the square in order to write the equation in factored form:

$(x^2 - 4x + 4) + (y^2 - 8y + 16) + 11 = 20$……..(2 points)

Then equation of the circle is:

$(x - 2)^2 + (y - 4)^2 = 9 = 3^2$…….……(1 point)

Correct graph of the circle (see figure labeled **KEY** to the right)……..(1 point)



Scoring Guide:

The question is worth 4 Points.

Partial credit may be given. Subtract 1 point per mistake in the equation or graph.

3 out of 4 indicates Mastery

Total Summative Assessment Points: 12

Mastery at: 9/12