Curriculum Sample Template – 8 Pages Max. (12 pages for integrated ELA sample)

Grade Level	12	**Content Area**	Mathematics
Course Title (grades 9-12 Only)	Pre-Calculus		
Expected Prior Knowledge *List the knowledge/skills mastered earlier in the year that are foundational to the mastery of the Required Standard(s).*	Students will already know how to: • Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G-SRT.C.6) • Explain and use the relationship between the sine and cosine of complementary angles. (G-SRT.C.7) • Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.(F-TF.A.1) • Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (F-TF.A.2)		
Alignment to Program of Instruction *Describe how the methods of instruction found in this sequence of lessons align to the Program of Instruction described in the application package.*	This lesson aligns to the program of instruction by using scaffolding techniques like modeling and guided practice. Students are gradually released to do the work on their own. These lessons are also sheltered by pre-teaching vocabulary and having students write the vocabulary in their Math Glossary. Students will also be engaged in active learning as they explore the unit circle on their own to gain mastery of the patterns revealed by the unit circle.		
Standard Number* and Description *The standard number and description of the Required Standard(s) being instructed and assessed to mastery in the curriculum sample.*	F-TF.A.3 – Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number.		
Materials/Resources Needed *List all items the teacher and students will need for the entire sequence of instruction (excluding common consumables)*	Student's Math Glossary Student's Math Journal Appendix: see endnote		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Cluster, Standard (e.g., 6.EE.B.7). For HS Math, use Conceptual Category-Domain, Cluster, Standard (e.g., A-REI.C.6).*

Lesson (add as needed)	**Instructional Strategies -** *List and describe teacher actions, lesson by lesson, which will provide students with opportunities to engage the grade-level rigor defined by the Required Standard(s). Indicate alignment of instructional strategies to specific component(s) or content/skill(s) of the Required Standard/component.*	**Student Activities -** *Describe the activities, lesson by lesson, in which students will engage to allow them to master the grade-level rigor defined by the Required Standard(s). Indicate alignment of student activities to specific component(s) or content/skill(s) of the Required Standard/component.*
1	**1. Anticipatory Set –** On the board: "Pop Rocks Quiz! What does SOH CAH TOA stand for?" The teacher will give the students an opportunity to remember, and then will call on students randomly to give them an opportunity to answer. Students who answer correctly will get some Pop Rocks® candy. **2. Learning Objective –** SWBAT find the sine, cosine, and tangent of an angle, given a right triangle with at least two side lengths known using the acronym SOH CAH TOA and use dimensional analysis to convert degrees to radians. **3. Instructional Input –** The teacher will remind students about the sine, cosine, and tangent ratios, and how they are calculated and used in real world situations. (Like indirect measurement, physics, etc.) The teacher will then introduce the unit of radians as another way to measure angles. The teacher will ask students to put the word "radian" in their Math Glossary as: A way of measuring angles based on the ratio of the radius to the circumference of a circle so that an arc length that is equal to the radius of the circle has an angle measure of 1 radian (1 rad.) Then the teacher will lead the students though the logic that since we know that the whole circumference of a circle is 2π times the radius then there are 2π radians of angle in a whole circle, or in other words 2π radians = 360° and π radians = 180°. The teacher will then introduce the abbreviation rad for radians, and the conversion factor 180° = π rad. The teacher will then remind students how to do dimensional analysis. **4. Modeling/Check for Understanding** – The teacher will do a conversion of degrees to radians on the board, and then a radians to degrees conversion and answer questions that students have. The teacher will then put a few problems on the board for the students to work out on their own, including some right triangles with known side lengths and angles given in radians. The teacher will circulate the room and assess the students' ability to convert from degrees to radians, and calculate sine, cosine and tangent. **5. Independent Practice –** The teacher will give the students a sheet of practice problems to and ask them to work them out on their own and turn them in as a formative assessment.	1. Students will write their answer to the Pop Rocks Quiz in their notes and answer when called upon. 2. Students will write the date and the learning objective in their notebook. 3. Students will take notes and ask questions as necessary to understand the information the teacher is conveying. Students will enter Radian in their Math Glossary. 4. Students will work the problems out and ask questions of the teacher as necessary to make sure they are doing the problems correctly. 5. Students will work the problems out independently and turn them in as a formative assessment. **Alignment to content/skills of Required Standard:** This lesson provides the foundational skills of being fluent with radians, converting between degrees and radians, and calculating the sine, cosine, and tangent geometrically, required to be able to master the required standard.

2	**1. Anticipatory Set –** On the board: A unit circle with an equilateral triangle (ΔABC) with a line segment from $\angle A$ to the midpoint of $\overline{BC}$ (See Figure A in the appendix.) and the directions: "The circle graphed here is a unit circle (centered at the Origin with radius =1,) points B and C are on the circle and point D is the midpoint of $\overline{AB}$. Show that ΔABC is an equilateral triangle and that $\overline{CD} \perp \overline{AB}$." The teacher will give the students time to work on this. After she feels that the students are finished she will lead a class discussion to make sure that all students understand that they have just created a right triangle on the unit circle. **2. Learning Objective –** SWBAT use special triangles to determine geometrically the values of sine, cosine, and tangent for π/3, π/4 and π/6. **3. Instructional Input –** The teacher will demonstrate on the board how, now that they know ΔACD (from the anticipatory set) is a 30-60-90 triangle, they can determine the sine, cosine, and tangent of 60° and 30° using what they know about the unit circle and the Pythagorean Theorem. The teacher will then demonstrate how a similar geometric proof can be constructed using the known properties of an isosceles right triangle (45-45-90), the Pythagorean Theorem, and the unit circle to determine the sine, cosine, and tangent of 45°. Then the teacher will demonstrate the correlation of the radian measures π/3, π/4, and π/6 to 60°, 45°, and 30° respectively both geometrically on the unit circle, and algebraically using dimensional analysis. **4. Modeling**– The teacher will demonstrate to the students how they can use the properties of special triangles, both on the unit circle and not, to solve different kinds of problems involving sine, cosine and tangent. **5. Guided Practice –** The teacher will give the students some problems to practice in class. The teacher will circulate the room and answer questions, and collect data on what students understand and what they don't understand.	1. Students will write out the reasoning that leads to the conclusion that ΔABC is an equilateral triangle and that $\overline{CD} \perp \overline{AB}$. (Two of the given legs of the triangle are a radius of the unit circle which means they are the same length [by the definition of a circle] which means that the triangle is isosceles. We know that the two unknown angles must be equal, since the triangle is isosceles, and that all three angles must sum to 180° so; 2x+60 = 180. Solving this yields x=60; since the degree measure of all three angles is the same, the triangle must be equilateral, and have sides all equal to 1. From the known properties of an equilateral triangle: there are three lines of reflective symmetry in an equilateral triangle each joining the vertex with the midpoint of its opposite side. Since $\overline{CD}$ goes through the midpoint of $\overline{AB}$, $\overline{CD} \perp \overline{AB}$.) 2. Students will write the date and the learning objective in their notebook. 3. Students will take notes and ask questions as necessary to understand the information the teacher is conveying. 4. Students will follow along as the teacher models the skill and record the sample in their notebook. 5. Students will attempt the problems given by the teacher and ask clarifying questions to make sure they know how to do the skill. **Alignment to content/skills of Required Standard:** This lesson directly aligns with the required standard because it teaches students how to use special triangles to calculate sine, cosine, and tangent for π/3, π/4 and π/6.
3	**1. Anticipatory Set –** On the board: "Draw a unit circle and identify and label all of the angles that are π/6 rad away from any axis by drawing a radius at that angle and a point where the radius intersects the unit circle. (Hint: there are 8.) Then find the sine and cosine of the angles in the first quadrant." After giving the students sufficient time, the teacher will put the answers on the board. (Answers: π/6, π/3, 2π/3, 5π/6, 7π/6, 4π/3, 5π/3, 11π/6, sin π/6 = 1/2,sin π/3 = $\sqrt{3}/2$, cos π/3 = 1/2, cos π/6 = $\sqrt{3}/2$)) **2. Learning Objective –** SWBAT use the unit circle to express the values of sine, cosine, and tangent for π-*x*, π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number. **3. Instructional Input –** The teacher will add the ordered pair for each of the points in the first quadrant and ask students what they notice. The	1. Student s will draw a unit circle and identify and label all of the angles that are π/6 rad away from any axis by drawing a radius at that angle and a point where the radius intersects the unit circle and give the sine and cosine for the angles in the first quadrant. 2. Students will write the date and the learning objective in their notebook. 3. Students will take notes and ask questions as necessary to understand the information the teacher is conveying.

	students will notice (on their own or with some coaching by the teacher) that the x-coordinate is always the cosine of the given angle, and the y-coordinate is always the sine of the given angle. (This will plant a seed for when they learn polar coordinates, take physics, and take calculus.) The students will also notice (on their own or with the teachers help) that the two values, 1/2 and $\sqrt{3}/2$, repeat themselves for each angle around the circle even though they switch between x and y and are negative or positive. The teacher will explain that this is because in each case, those angles make the same special right triangle (π/6-π/3-π/2 or 30°-60°-90°). The teacher will then instruct the students to put the word "reference angle" in their Math Glossary as "the acute angle made by the x-axis and the terminal side of the angle. The sine, cosine and tangent are the same for the angle and the reference angle, but might be negative." The teacher will demonstrate how the sine cosine and tangent are the same on the given angle as the reference angle. The teacher will then show how they will know whether the sine, cosine, or tangent is negative based on what quadrant it's in both by reviewing in which quadrants x and y are positive and negative and also by the mnemonic device "All Students Take Calculus" (Taking the quadrants in order: All are positive, Sine is positive, Tangent is positive, Cosine is positive. The CAST rule is a simple mnemonic that will help students remember the correct sign.) The teacher will then recommend that the students memorize sine, cosine and tangent values for the two special right triangles. For if they do, they will be able to quickly and easily solve trigonometry problems in any quadrant. **4. Modeling/Check for Understanding** – The teacher will ask the students to add π/4, 3π/4, 5π/4, and 7π/4 to their unit circle and include the ordered pair for their points. The final Product will look like Figure B in the Appendix for Pre-Calculus. **5. Guided Practice –** The teacher will put a few angles on the board that include multiples of π/4, π/3, and π/6 and occur in all four quadrants, and ask students at random for the next step or how they will find the answer. The teacher can also assign students to do one and circulate the room to answer questions and help students to do the next steps. **6. Independent Practice –** The teacher will give the students practice problems to find the sine, cosine, and tangent of angles that are multiples of π/4, π/3, and π/6 and occur in all four quadrants to do independently and turn in as a formative assessment.	4. Students will add π/4, 3π/4, 5π/4, and 7π/4 to their unit circle and include the ordered pair for their points. 5. Students will follow along as the teacher does the example problem and attempt to do the problems the teacher assigned and ask clarifying questions to ensure they understand. 6. Students will work the problems out independently and turn them in as a formative assessment. **Alignment to content/skills of Required Standard:** This lesson aligns to the required standard by teaching students that the sine, cosine, and tangent of the angle are the same for all angles that have the same reference angle, with the exception that in some cases they may be negative. This directly aligns with the second part of the standard: use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number.
S.A.	*Provide an opportunity for the student to complete the summative assessment items which is clearly separate from instruction and guided or independent practice, and in which the student is assessed*	An optional review period may be conducted before the end of unit test. Students will take an end of unit test immediately following the unit. The test will be free response and taken individually and take about 1 class

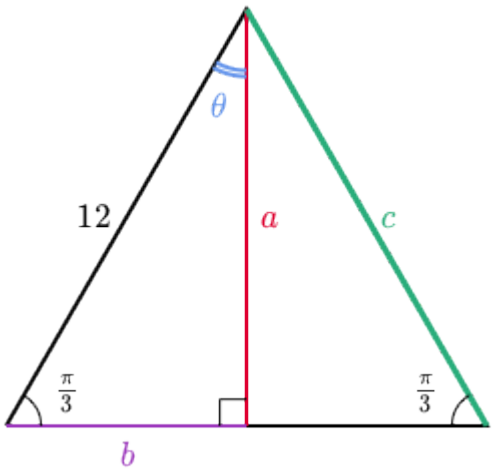
	independently. In the Student Activities column, provide a brief description of the summative assessment (S.A.) items that will allow students to demonstrate mastery of the rigor of the Required Standard(s)/components, and the context in which the items will be administered.	period. Students will have the use of a scientific calculator: either their own, or one provided to them. Smart phones will not be allowed to be used as a calculator. It will a closed note test. The skills addressed in the standard will be assessed a second time (at least) in a Quarterly Common Assessment at the end of the quarter.

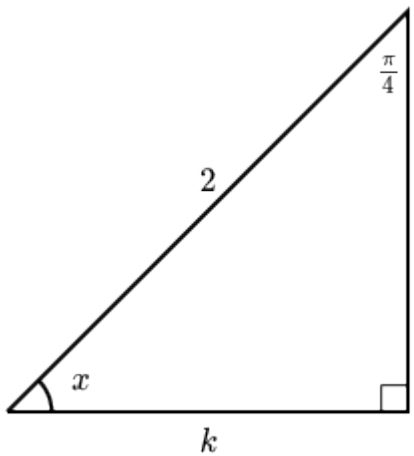
Summative Assessment Items and Scoring: *Below, provide at least three Summative Assessment Items that together accurately measure student mastery of the application of the content and/or skills as defined by the grade-level rigor in the Required Standard(s). For each assessment item, provide an answer key or scoring rubric which clearly describes how assessment items are to be scored and how points will be awarded, such that mastery of the application of the content and/or skills as defined by the grade-level rigor in the Required Standard is clearly demonstrated by an acceptable score.*

Summative Assessment Item 1 (clearly identify answer key/scoring rubric, and acceptable score):

Use the figures below to find the values of the following expressions.

Don't round. All answers should be exact. Angle measures should be given in *radians*.





1. θ =
2. x =
3. a =
4. b =
5. c =
6. k =
7. $\sin\frac{\pi}{3} =$
8. $\cos\frac{\pi}{6} =$
9. $\tan\theta =$
10. $\sin x =$
11. $\cos\frac{\pi}{4} =$
12. $\tan x =$

KEY

1. θ = π/6 (sum of all angles is π); 2. x = π/4 (sum of all angles is π); 3. a = $6\sqrt{3}$ (in 30-60-90 triangle long side is short side times $\sqrt{3}$); 4. b = 6 (in 30-60-90 triangle short side is half the hypotenuse); 5. c = 12 (in equilateral triangle all side are equal.); 6. k = $\sqrt{2}$ (in 45-45-90 triangle legs are $\frac{\sqrt{2}}{2}$

times the hypotenuse); 7. $\sin\frac{\pi}{3} = \frac{opposite}{hypotenuse} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$; 8. $\cos\frac{\pi}{6} = \frac{adjacent}{hypotenuse} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$; 9. $\tan\theta = \frac{opposite}{adjacent} = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (Both $\frac{1}{\sqrt{3}}$ *and* $\frac{\sqrt{3}}{3}$ are acceptable); 10. $\sin x = \frac{opposite}{hypotenuse} = \frac{\sqrt{2}}{2}$; 11. $\cos\frac{\pi}{4} = \frac{adjacent}{hypotenuse} = \frac{\sqrt{2}}{2}$; 12. $\tan x = \frac{opposite}{hypotenuse} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

Scoring Guide

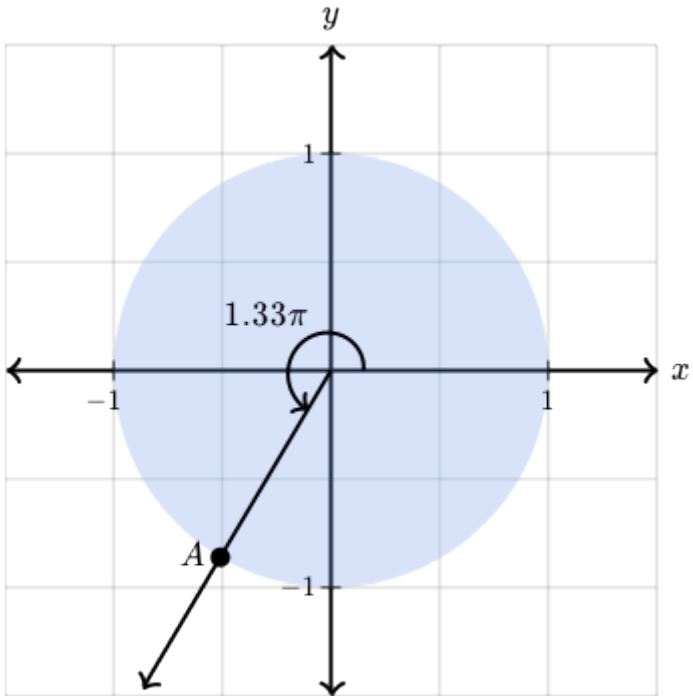
This question is worth 12 points. Partial credit may be given: 1 point for each correct answer.

Summative Assessment Item 2 (clearly identify answer key/scoring rubric, and acceptable score):

Use the figure to the right to answer the following question.

Which of the expressions below are equivalent to the y-coordinate of the point *A*?

Circle all that apply, and give a brief explanation of why it's equivalent.



a. sin (9.33π)

b. sin (5.33π)

c. cos (3.33π)

d. sin (3.33π)

KEY

a, b, and d are equivalent because they are 8π, 4π, and 2π respectively bigger than 1.33π.

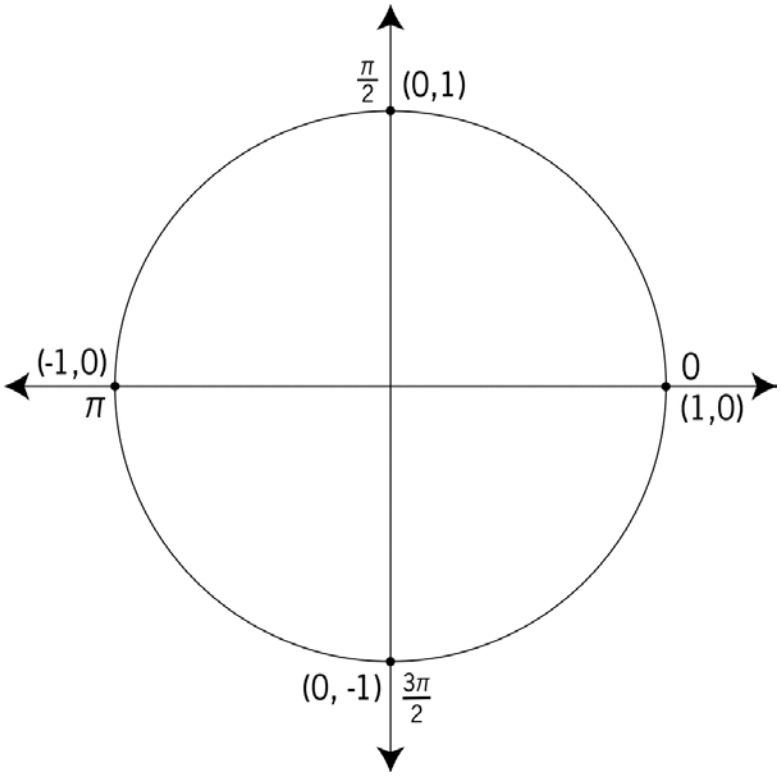
Scoring Guide

This question is worth 6 points.

Partial credit of 1 point each may be given for circling the correct expression without giving an explanation.

Summative Assessment Item 3 (clearly identify answer key/scoring rubric, and acceptable score): (add as needed):

Give the value of the expressions below. You may use the unit circle to the right to help you out.



a. $\sin\frac{5\pi}{3} =$

b. $\cos\frac{5\pi}{6} =$

c. $\tan\frac{5\pi}{4} =$

d. $\sin\frac{3\pi}{4} =$

e. $\cos\frac{13\pi}{4} =$

f. $\tan\frac{2\pi}{3} =$

KEY

a. $\sin\frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$

b. $\cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$

c. $\tan\frac{5\pi}{4} = 1$

d. $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

e. $\cos\frac{13\pi}{4} = \frac{-\sqrt{2}}{2}$

f. $\tan\frac{2\pi}{3} = -\sqrt{3}$

Scoring Guide

This problem is worth 6 points.
Partial credit may be given, 1 point for each correct answer.

Total Points = 24
Mastery = 17/24

Appendix for Pre-Calculus

Figure A:

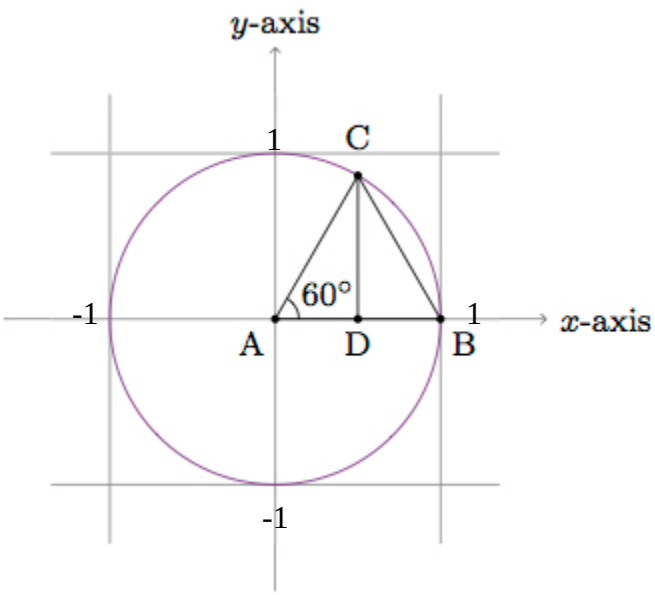
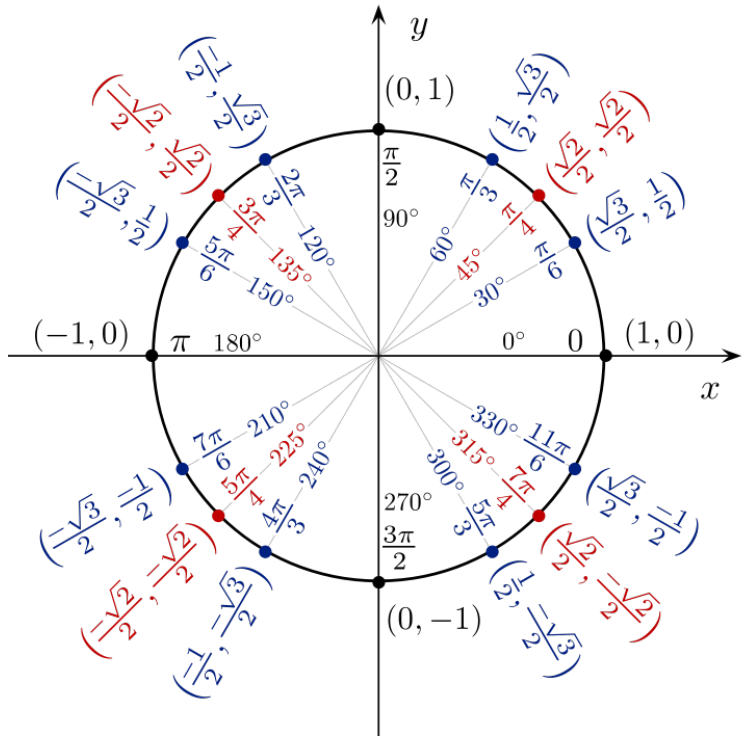



Figure B: