Curriculum Sample Template – 8 Pages Max. (12 pages for integrated ELA sample)

Grade Level	11	**Content Area**	Mathematics
Course Title (grades 9-12 Only)	Algebra II		
Expected Prior Knowledge *List the knowledge/skills mastered earlier in the year that are foundational to the mastery of the Required Standard(s).*	Students will already be able to: • Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.A.1)		
Alignment to Program of Instruction *Describe how the methods of instruction found in this sequence of lessons align to the Program of Instruction described in the application package.*	These lessons have been scaffolded to make learning this abstract concept a smoother process. Each lesson features a clear learning objective at the beginning of the lesson, along with some activation of prior knowledge. The vocabulary is taught along with the lesson and incorporated into the lesson so that student have multiple opportunities to engage with the vocabulary in the context of the content. The students are given multiple examples of what they are expected to be able to do both in the instructional input phase of the lesson and in the modeling or guided practice phase. The lessons also progress from a place where all students should feel comfortable towards the more abstract and advanced content in a gradual manner to ensure that most students will follow along. In the process the teacher is connecting what students learned previously to what they are learning now and making explicit the analogous concepts. In order to make learning a more active process for students; the students are encouraged to use pictures and/or examples in their Math Glossary.		
Standard Number* and Description *The standard number and description of the Required Standard(s) being instructed and assessed to mastery in the curriculum sample.*	A-APR.C.4. – Prove polynomial identities and use them to describe numerical relationships.		
Materials/Resources Needed *List all items the teacher and students will need for the entire sequence of instruction (excluding common consumables)*	Student's Math Glossary Student's Math Journal Appendix – See Endnote		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Cluster, Standard (e.g., 6.EE.B.7). For HS Math, use Conceptual Category-Domain, Cluster, Standard (e.g., A-REI.C.6).*

Lesson (add as needed)	**Instructional Strategies -** *List and describe teacher actions, lesson by lesson, which will provide students with opportunities to engage the grade-level rigor defined by the Required Standard(s). Indicate alignment of instructional strategies to specific component(s) or content/skill(s) of the Required Standard/component.*	**Student Activities -** *Describe the activities, lesson by lesson, in which students will engage to allow them to master the grade-level rigor defined by the Required Standard(s). Indicate alignment of student activities to specific component(s) or content/skill(s) of the Required Standard/component.*
1	**1. Anticipatory Set –** Question on board: "How do you know when two things are equal? Give at least three examples of things that look different, but are equal." The teacher will give the students 5 minutes to answer the question then lead a discussion about what equality is. Students will enter "equality" into their Math Glossary. **2. Learning Objective –** SWBAT use written "rules" to convert a polynomial expression to an equivalent form using algebraic manipulation. **3. Instructional Input –** The teacher will show, using the example of $(x+3)^2 = x^2 + 6x + 9$ by deriving $x^2 + 6x + 9$ from $(x+3)^2$ using the distributive property, that those two expressions are equal because, no matter what value of x is used, those two expressions result in the same value. The teacher will then demonstrate, using the example $(x+a)^2 = x^2 + 2ax + a^2$, by deriving $x^2 + 2ax + a^2$ from $(x+a)^2$ using the distributive property, that those two expressions are equal because, no matter which value of x and a we choose, the expressions result in the same value. The teacher will use several values of x and a, and the help of the students, to verify that claim to the students' content. The teacher will then explain that when we use the rules of algebra to convert one expression into another equivalent expression, that's called algebraic manipulation. The teacher will instruct the students to add "algebraic manipulation" to their Math Glossary. The teacher will then present the rules of algebraic manipulation for the class to write in their notes and ask questions as necessary. **4. Check for Understanding –** The teacher will give the students some practice problems that assess their ability to use algebraic manipulation. The teacher will assess the students by walking around and observing them work out the problems, what questions are asked during the work time, and the level of mastery of the rules provided. The teacher may decide than an extra day of practice is required before moving on. **5. Independent Practice –** The teacher will give the students additional practice problems to work out on their own (or as a class the following day.)	1. Students will enter "equality" into their Math Glossary as: a relationship between two quantities or mathematical expressions in which the two have the same value or represent the same thing even if they look different. 2. Students will enter the date and the learning objective in their Math notebook. 3. Students will take notes, ask questions to clarify, and enter "algebraic manipulation" into their glossary. 4. Students will attempt to work out the problems presented, and ask questions to clarify their understanding of algebraic manipulation. 5. Students will do the assigned problems and turn them in as a formative assessment grade. **Alignment to content/skills of Required Standard:** This lesson prepares student with the skills they will need in order to prove polynomial identities.

2	**1. Anticipatory Set –** Question on board: "Show the steps necessary to change $(x-5)^2$ into $x^2-10x+25$." **2. Learning Objective –** SWBAT use written rules and examples of algebraic manipulation to prove polynomial identities. **3. Instructional Input –** The teacher will show students how to line up their equals signs and do one step on each line to show their steps while doing algebraic manipulation. Then the teacher will inform the students that if we can use the rules of algebraic manipulation to show that two expressions are equal to each other for every possible value of its variables, then we call those two expressions an identity. The teacher will then direct the students to enter "identity" in their Math Glossary. The teacher will then explain how proving a polynomial identity just means to show all the steps required to get from one version of the polynomial to the other correctly. The teacher will reiterate the importance of using algebraic manipulation correctly. In order to illustrate this, the teacher will do the "proof" that 1=2. The teacher will ask the students to point out the step that was incorrect in the proof. The teacher will then reassure students that they already know the algebra to be able to prove these polynomial identities and that as long as they persevere in solving them, and learn from their errors, they will be just fine. **4. Guided Practice –** The teacher will put an equation on the board: $(a+b^2)(a^2-3)=a^3+a^2b^2-3(a+b^2)$ and will guide the students as they show the proof by cold calling on students for next steps. The teacher will use the guided practice as a formative assessment based on how well students know the steps, and questions they ask. **5. Independent Practice –** The teacher will give the students several problems to prove to practice their skills.	1. Students will show all steps as they square the binomial. 2. Students will enter the date and the learning objective in their Math notebook. 3. Students will take notes, ask questions to clarify, and enter "identity" into their glossary. 4. Students will follow along and state the next step for the teacher when called upon. 5. Students will do the assigned problems and turn them in as a formative assessment grade. **Alignment to content/skills of Required Standard:** This lesson aligns with the standard by teaching students how to prove polynomial identities.
3	**1. Anticipatory Set –** Question on board: Write an expression describing the pattern 3, 7, 11, 15, … using n so that n is the nth term or the series. (Answer: 4n-1) Depending on how students respond to this the teacher may walk them through the process or call on students to help. **2. Learning Objective –** SWBAT describe numerical relationships using polynomial identities. **3. Instructional Input –** The teacher will explain how algebraic expression can often be used to describe mathematical patterns. The teacher will show some examples of how numerical patterns can be described by an algebraic expression. The teacher will also point out the similarities of a function to the descriptions of patterns as this will help them in later classes as they learn about mathematical series. The teacher will also demonstrate how the expression $(x^2+y^2)^2=(x^2-y^2)^2+$	1. Students will attempt to express the pattern as an algebraic expression using n so that n is the nth term of the series. 2. Students will enter the date and the learning objective in their Math notebook. 3. Students will take notes, and ask questions to clarify. 4. Students will input several positive integer values of x and y and experiment with using the same value and 0 to explain why those conditions don't work to generate a Pythagorean Triple. 5. Students will prove the identity $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$,

	$(2xy)^2$ can be used to generate Pythagorean Triples. The teacher will ask students if they can see the Pythagorean Theorem in the expression. The teacher will show how no matter what positive integer values of x and y they use, the result will always be a Pythagorean Triple. The teacher will also explain the limitations of the equation that neither x nor y can be 0, nor can they be the same. **4. Check for Understanding** – The teacher will then ask students to try several sets of integers including some that include a 0 and are the same. And ask the students why those won't work. (Because the result is a side length of 0, which isn't a triangle at all.) **5. Independent Practice** – The teacher will then ask the students to prove the identity $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$ individually as a formative assessment.	and turn in their work as a formative assessment grade. **Alignment to content/skills of Required Standard:** In this lesson students learn about describing mathematical patterns using algebraic expressions.
S.A.	*Provide an opportunity for the student to complete the summative assessment items which is clearly separate from instruction and guided or independent practice, and in which the student is assessed independently. In the Student Activities column, provide a brief description of the summative assessment (S.A.) items that will allow students to demonstrate mastery of the rigor of the Required Standard(s)/components, and the context in which the items will be administered.*	An optional review period may be conducted before the end of unit test. Students will take an end of unit test immediately following the unit. The test will be free response and taken individually and take about 1 class period. Students will have the use of a scientific calculator: either their own, or one provided to them. Smart phones will not be allowed to be used as a calculator. It will a closed note test. The skills addressed in the standard will be assessed a second time (at least) in a Quarterly Common Assessment at the end of the quarter.

Summative Assessment Items and Scoring: *Below, provide at least three Summative Assessment Items that together accurately measure student mastery of the application of the content and/or skills as defined by the grade-level rigor in the Required Standard(s). For each assessment item, provide an answer key or scoring rubric which clearly describes how assessment items are to be scored and how points will be awarded, such that mastery of the application of the content and/or skills as defined by the grade-level rigor in the Required Standard is clearly demonstrated by an acceptable score.*

Summative Assessment Item 1 (clearly identify answer key/scoring rubric, and acceptable score):

Prove that $5k^2+5=(k-2)^2+(2k+1)^2$.

KEY:

$(k-2)^2+(2k+1)^2 \quad =(k-2)(k-2)+(2k+1)(2k+1)$ (1 Point)

$=k^2-4k+4+4k^2+4k+1$ (1 Point)

$=5k^2+5$ (1 Point)

Scoring guide: Question is worth 3 points.

Partial credit may be given according to rubric above.

Summative Assessment Item 2 (clearly identify answer key/scoring rubric, and acceptable score):

Given below is an attempt at proving a polynomial identity.

Are the proof and the identity valid? If not, what is the invalid step in the proof and why is it invalid?

	$(2x^2 - 18)(x + 3)(x - 3)$
Step 1	$= 2(x^2 - 9)(x^2 - 3x + 3x - 9)$
Step 2	$= 2(x^2 - 9)(x^2 - 9)$
Step 3	$= 2(x^2 - 9)^2$
Step 4	$= 2(x^4 - 81)$
Step 5	$= 2x^4 - 162$

KEY:
Step 4 is invalid because the binomial $(x^2 - 9)^2$ is improperly squared.

Scoring Guide:
The question is worth 2 points.

1 point may be awarded if the student correctly identifies the invalid step but either fails to give a reason why, or the reason is incorrect.

Summative Assessment Item 3 (clearly identify answer key/scoring rubric, and acceptable score): (add as needed):

Write an identity that can explain the pattern below and is true for any integer, n, then prove the identity.

$2^2 - 1^2 = 3$

$3^2 - 2^2 = 5$

$4^2 - 3^2 = 7$

$5^2 - 4^2 = 9$

KEY:

$(n+1)^2 - n^2 = 2n + 1$ (3 Points)

- 1 point for an identity that explains the pattern but has $n_1 \neq 1$.

For example, $(n+2)^2 - (n+1)^2 = 2n + 3$ is a valid identity, but in this case n_1=0.

Simplify: (2 points)

$$\begin{aligned}(n+1)^2 - n^2 &= (n+1)(n+1) - n^2 \\ &= n^2 + 2n + 1 - n^2 \\ &= 2n + 1\end{aligned}$$

Scoring Guide:

This question is worth 5 points.

Partial credit may be awarded by subtracting 1 point for each mistake.

Total Points = 10

Mastery = 7/10

Endnote: Rules of Algebraic Manipulation

1. The Communicative Properties of Addition and Multiplication:

   It doesn't matter what order you add or multiply numbers in, the result is always the same.

   $a + b = b + a; \quad and \quad ab = ba$

2. The Distributive Property:

   $a(b + c) = ab + ac; \quad and \quad fg + fh = f(g + h)$

   Remember you can factor out common factors.

   Remember FOIL (First Outer Inner Last) $(a + b)(c + d) = ac + ad + bc + bd$

   Remember $(a + b)^2 = (a + b)(a + b)$

   <u>DO NOT</u> make this common error $\boxed{\cancel{(a + b)^2 = a^2 + b^2}}$

3. Identity and Properties: (These will come in VERY handy.)

   Anything plus 0 is itself. $a + 0 = a$

   Anything times 1 is itself. $1 * a = a$

   Anything divided by 1 is itself. $a \div 1 = a \quad or \quad \frac{a}{1} = a$

   Anything divided by itself is 1. $\frac{a}{a} = 1$

   Any quantity plus its negative is 0. $a + (-a) = 0$

4. Inverse operations: (Also handy.)

   Addition and Subtraction undo each other. $(a + b) - a = b \quad and \quad (a - b) + b = a$

   Multiplication and division undo each other. $\frac{ab}{a} = b \quad and \ \frac{a}{b} * b = a$

   Powers and roots undo each other. $\left(\sqrt{a}\right)^2 = a; \ \left(\sqrt[n]{a}\right)^n = a \quad and \quad \sqrt{a^2} = a; \ \sqrt[n]{a^n} = a$

5. Treat all negatives as multiplication by -1:

   $-a = (-1)(a) \qquad a - b = a + (-1)(b)$

   $-(a + b) = (-1)(a + b) = -a - b$

6. Properties of Equality:

   Add the same thing to both sides of the equation to maintain equality.

   Example: $If\ a = b,\ then\ a + c = b + c$

   Subtract the same thing from both sides of the equation to maintain equality.

   Example: $If\ a = b,\ then\ a - c = b - c$

Multiply the same thing to both sides of the equation to maintain equality.

Example: $If\ a = b,\ then\ c * a = c * b$

Divide both side by the same thing to maintain equality.

Example; $If\ a = b,\ then\ \frac{a}{c} = \frac{b}{c}$

False Proof that 1=2

Step 1	Let a = b	$a = b$
Step 2	Multiply both sides by a	$a^2 = ab$
Step 3	Subtract b^2 from both sides	$a^2 - b^2 = ab - b^2$
Step 4	Factor both sides	$(a + b)(a - b) = b(a - b)$
Step 5	Divide both sides by (a – b)	$(a + b) = b$
Step 6	Observe that a = b	$(b + b) = b$
Step 7	Combine like terms on the left	$2b = b$
Step 8	Divide both sides by b	2 = 1