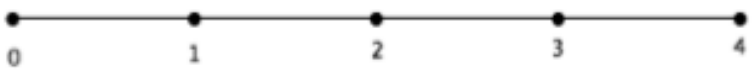
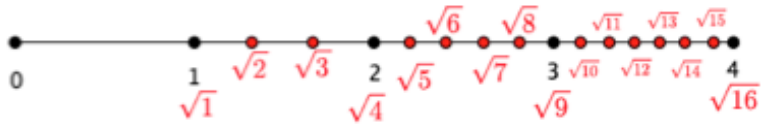
Grade Level	Eighth Grade	**Content Area**	Mathematics
Course Title (grades 9-12 Only)	N/A		
Expected Prior Knowledge *List the knowledge/skills mastered earlier in the year that are foundational to the mastery of the Required Standard(s).*	Scholars should understand perfect squares and how to calculate their inverse roots. Scholars must be able to correctly calculate products of decimals with hundredths place values. Scholars should already have experience with comparing and ordering rational numbers, including the use of a number line. Scholars have already used the Pythagorean Theorem to find a missing side length of a right triangle but left non-perfect squares in root notation.		
Alignment to Program of Instruction *Describe how the methods of instruction found in this sequence of lessons align to the Program of Instruction described in the application package.*	Arizona Career and College Readiness Core math standards form the basis for The Grande Innovation Academy's math curriculum. Scholars will learn standards using Arizona Career and College Readiness Core strands and the knowledge and skills scholars learn will build from year to year. The Grande Innovation Academy will use EngageNY math modules along with manipulatives to build conceptual understanding. Additionally, The First in Math online program will be used for developing automaticity with math facts to increase mathematical fluency. Each year at The Grande Innovation Academy scholars in grade Kindergarten through eighth will participate in a school wide Math Olympics program to generate motivation for math fluency.		
Standard Number* and Description *The standard number and description of the Required Standard(s) being instructed and assessed to mastery in the curriculum sample.*	**8.NS.A.2** *Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*		
Materials/Resources Needed *List all items the teacher and students will need for the entire sequence of instruction (excluding common consumables)*	Individual whiteboards Dry erase markers Interactive math journals Number lines including decimal place value Document camera		

* ***Standard Number****: For English Language Arts (Reading and Writing), use Grade, Strand, Standard (e.g., 3.RI.2). For K-8 Math, use Grade, Domain, Cluster, Standard (e.g., 6.EE.B.7). For HS Math, use Conceptual Category-Domain, Cluster, Standard (e.g., A-REI.C.6).*

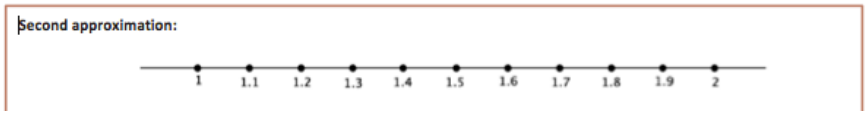
Lesson (add as needed)	**Instructional Strategies -** *List and describe teacher actions, lesson by lesson, which will provide students with opportunities to engage the grade-level rigor defined by the Required Standard(s). Indicate alignment of instructional strategies to specific component(s) or content/skill(s) of the Required Standard/component.*	**Student Activities -** *Describe the activities, lesson by lesson, in which students will engage to allow them to master the grade-level rigor defined by the Required Standard(s). Indicate alignment of student activities to specific component(s) or content/skill(s) of the Required Standard/component.*
1	**Objective**: Scholars use rational approximations to estimate the square roots of non-perfect squares to the nearest integer. **Anticipatory Set:** Teacher instructs scholars to draw a number line that includes integers as well as other rational numbers. After scholars complete their number lines, display some on the document camera and lead class discussion of what types of numbers are on the number line and how scholars know where they belong. **Direct Instruction/Guided Practice:** Teacher makes scholars aware there are other numbers on the number line between the integers. They are called square roots. Some of the square roots are equal to whole numbers, but most lie between the integers on the number line. A positive number whose square is equal to a positive number b is denoted by the symbol √b. The symbol √b automatically denotes a positive number (e.g., √4 is always 2, not -2). The number √b is called a positive square root of b. • Take a number line from 0 to 4:  • Place the numbers √1, √4, √9, and √16 on the number line and explain how you knew where to place them. • Place the numbers √2 and √3 on the number line. Be prepared to explain your reasoning. • Place the numbers √5, √6, √7, √8, √10, √11, √12, √13, √14, and √15 on the number line. Be prepared to explain your reasoning. Our work on the number line shows that there are many more square root numbers that are not perfect squares than those that are	**Anticipatory Set:** Scholars draw their number lines to include integers and rational numbers like fractions and decimals. Scholars participate in class discussion of correct placement of rational numbers between integers. **Direct Instruction/Guided Practice:** Scholars take notes including example problems in interactive math journals. Scholars respond to teacher questioning during checks for understanding.  Scholars should reason that the numbers $\sqrt{2}$ and $\sqrt{3}$ belong on the number line between $\sqrt{1}$ and $\sqrt{4}$. They could be more specific by saying that if you divide the segment between integers 1 and 2 into three equal parts, then $\sqrt{2}$ would be at the first division and $\sqrt{3}$ would be at the second division and $\sqrt{4}$ is already at the third division, 2 on the number line. Given that reasoning, students should be able to estimate the value of $\sqrt{2} \approx 1\frac{1}{3}$. Similar reasoning should follow for the rest of the numbers.

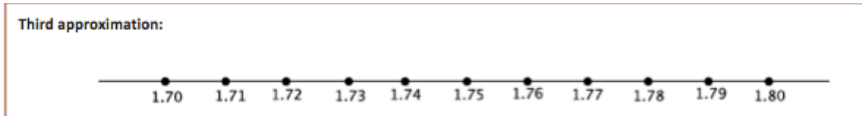
	perfect squares. On the number line above, we have four perfect square numbers and twelve that are not! After we do some more work with roots, in general, we will cover exactly how to describe these numbers and how to approximate their values with greater precision. For now, we will estimate their locations on the number line using what we know about perfect squares. **Independent Practice:** Teacher provides practice problems and time to work before engaging class discussion. During work time, teacher circulates and provides additional support as needed. During discussion, teacher looks for differences in valid reasoning and discusses those differences when appropriate. Determine the positive square root of the number given. If the number is not a perfect square, determine which integer the square root would be closest to. 1. $\sqrt{49}$ 2. $\sqrt{62}$ 3. $\sqrt{122}$ 4. $\sqrt{400}$ 5. $\sqrt{196}$ 6. $\sqrt{50}$ **Closure/Exit Ticket:** Teacher assesses scholar understanding based on exit ticket answers and adjusts instruction accordingly.	**Independent Practice:** Scholars work on practice problems and be prepared to share and support their conclusions with the whole group. 1. 7 2. Close to 8 3. Close to 11 4. 20 5. 14 6. Close to 7 **Closure/Exit Ticket:** Scholars complete exit ticket to demonstrate mastery. Place the following numbers on the number line below: $\sqrt{16}$, $\sqrt{9}$, $\sqrt{11}$, 3.5.
2	**Objective**: Scholars use rational approximation to calculate the decimal expansion of irrational numbers to the nearest hundredth and locate them on a number line. **Anticipatory Set:** Teacher poses the opening question to scholars: Place √28 on a number line. What decimal do you think √28 is equal to? Explain your reasoning. Lead a discussion where scholars share their reasoning as to the placement of √28 on the number line. Encourage scholars to critique the reasoning of others while evaluating their own arguments. Scholars will vote on the placement they think is most correct. **Direct Instruction/Guided Practice:** Teacher introduces concept of rational approximation and model problems. So far we have been able to estimate the size of a number like √3 by stating that it is between the two perfect squares √1 and √4,	**Anticipatory Set:** Scholars work with a partner or in small groups to answer the question and explain their thinking. During discussion, scholars will respectfully and appropriately critique one another's reasoning and ultimately vote on where they believe the number would be on the number line. **Direct Instruction/Guided Practice:** Scholars take notes of example problems in interactive math journals. Scholars respond to teacher questioning during checks for understanding.

meaning that √3 is between 1 and 2 but closer to 2. To be more precise, we can use rational approximation: using a sequence of rational numbers to get closer and closer to a given number to estimate the value of a number.

- To write the decimal expansion of √3 we first determine between which two integers the number √3 would lie on the number line. This is our first approximation. What are those integers?
- To be more precise with our estimate of √3, we now look at the tenths between the numbers 1 and 2. This is our second approximation. The question becomes, where exactly would √3 lie on this magnified version of the number line? Since we know that it would be closer to 2, we can use this rough estimate to try different squares of the tenths places until we find the closest value: $1.9^2 = 3.61$, $1.8^2 = 3.24$, $1.7^2 = 2.89$
- What do you think will need to be done to get an even more precise estimate of the number √3? We can go out another decimal place value and estimate to the nearest hundredth and then even further if needed. Let's try 1.75^2 to start = 3.0625 (too high). $1.74^2 = 3.0276$ (closer). $1.73^2 = 2.9929$
- This method of looking at successive intervals is what we call rational approximation. With each new interval we are approximating the value of the number by determining which two rational numbers it lies between.

Independent Practice: Teacher provides practice problems and time to work before engaging class discussion. During work time, teacher circulates and provides additional support as needed. During discussion, teacher looks for differences in valid reasoning and discusses those differences when appropriate.
1. Now that we know this process, approximate the value of √28 (from our opening activity) to the nearest hundredth.
2. Between which intervals of hundredths would √14 be located? Show your work.





Independent Practice: Scholars work on practice problems and be prepared to share and support their conclusions with the whole group.

1. Between 5.29 and 5.30, closer to 5.29

2. Between 3.74 and 3.75, closer to 3.74

	Closure/Exit Ticket: Teacher assesses scholar understanding based on exit ticket answers and adjusts instruction accordingly. • Explain how you would use rational approximation to estimate the value of an expression.	**Closure/Exit Ticket:** Scholars complete exit ticket to demonstrate understanding of concept.
3	**Objective**: Scholars use rational approximations to compare the size of irrational numbers and make comparative statements. **Anticipatory Set:** Teacher poses opening question and allows time for scholars to determine answer before leading discussion of class answers and explanations. • For what integer x is $\sqrt{x}$ closest to 3.17? Explain. **Direct Instruction/Guided Practice:** Teacher asks scholars to explain the difference between rational and irrational numbers. Scholar-created definition of irrational numbers is written into math journals. Teacher asks scholars to recall previous lesson on rational approximation and explains the process can also be used to make comparisons between numbers. Guided practice questions include: • Which number is greater, √50 or 319/45? Explain. • Which number is smaller, √38 or 154/25? Explain. • Which number is greater, √2 or 15/9? Explain. **Independent Practice:** Teacher provides practice problems and time to work before engaging class discussion. During work time, teacher circulates and provides additional support as needed. During discussion, teacher looks for differences in valid reasoning and discusses those differences when appropriate. 1. Place the following numbers at their approximate location on the number line: √25, √28, √30, √32, √35, √36. 2. Which number is greater, -√50 or -7? 3. Place the following numbers at their approximate location on the number line: √12, √16, 20/6, 3.53, 3 1/3.	**Anticipatory Set:** Scholars work in pairs to solve the given problem. Answers should involve solving $3.17^2 = 10.0489$ to see that x would be 10 in this case. Since $\sqrt{x}$ is the square root of x, then x^2 will give the integer that belongs in the square root. **Direct Instruction/Guided Practice:** Scholars include definition for irrational numbers in their own words in interactive math journals including examples, non-examples, and a mind sketch (picture). Scholars solve guided practice problems on whiteboards to show mathematical reasoning. Scholars respond to teacher questioning during checks for understanding. **Independent Practice:** Scholars work on practice problems and be prepared to share and support their conclusions with the whole group.

	Closure/Exit Ticket: Teacher assesses scholar understanding based on exit ticket answers and adjusts instruction accordingly. • How do we know if a number is rational or irrational? • Is the number √2 rational or irrational? Explain.	**Closure/Exit Ticket:** Scholars complete exit ticket to demonstrate understanding.
S.A.	**Objective**: Scholars apply rational approximation to identify and order irrational square roots. Teacher uses summative assessment items to evaluate scholar understanding and progress toward mastery.	Scholars are asked to complete word problems to identify integers on a number line to determine greater/lesser than numbers. They are asked to explain their answers. Scholars are seated at their tables to complete the assessment.

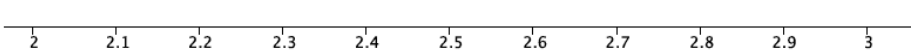
Summative Assessment #1

1. a) Between which two positive integers does $\sqrt{33}$ lie? Explain.
   b) For what integer x is $\sqrt{x}$ closest to 5.25? Explain.

Summative Assessment #2

2. Determine the greater number in each of the pairs (a)–(c) below.
   a. Which is greater? 8 or $\sqrt{60}$
   b. Which is greater? 4 or $\sqrt{26}$
   c. Which is greater? -7 or $-\sqrt{42}$

Summative Assessment #3

3. 

   a. Between which two labeled points on the number line would $\sqrt{5}$ be located? Locate and label it.
   b. Explain how you know where to place $\sqrt{5}$ on the number line.
   c. How could you improve the accuracy of your estimate?

Answer Key:

1. a) Between 5 and 6 because $5^2 = 25$ & $6^2 = 36$ b) $(5.25)^2 = 27.5625$ Since $\sqrt{x}$ is the square root of *x,* then x^2 will give the integer that belongs in the square root. 27.5625 is closest to the integer 28. *A score of three out of four points (see rubric) is necessary to show mastery (75%)*

2. a) 8 b) $\sqrt{26}$ c) $-\sqrt{42}$ *A score of three out of four points (see rubric) is necessary to show mastery (75%)*

3. a) Between 2.2 and 2.3 b) $2^2 = 4$ and $3^2 = 9$, so $\sqrt{5}$ is between 2 and 3 but closer to 2. Check intervals of tenths beginning with 2.1 to find it is between 2.2 and 2.3 because $2.2^2 = 4.84$ and $2.3^2 = 5.29$. c) To improve the accuracy, continue the method of rational approximation to determine which hundredths it fits between. Once hundredths is known, thousandths can be determined and so on. *A score of three out of four points (see rubric) is necessary to show mastery (75%)*

Summative Assessment Scoring:

A score of three out of four points is necessary to show mastery (75%) on each: Summative Assessment 1, Summative Assessment 2, and Summative Assessment 3.

A Progression Toward Mastery				
Standards Assessed	**STEP 1** **Little evidence of reasoning without a correct answer.** **(1 Point)**	**STEP 2** **Evidence of some reasoning without a correct answer.** **(2 Points)**	**STEP 3** **Evidence of some reasoning with a generally correct answer.** **(3 Points)**	**STEP 4** **Evidence of solid reasoning with a correct answer.** **(4 Points)**
Assessment 1 **8.NS.A.2**	Scholar does not attempt problem or writes answers that are incorrect for both parts (a)–(b).	Scholar is able to answer at least one of the parts (a)–(b) correctly. Scholar may or may not provide a weak justification for answer selection.	Scholar is able to answer both parts (a)–(b) correctly. Scholar may or may not provide a justification for answer selection. Explanation includes some evidence of mathematical reasoning.	Scholar correctly answers both parts (a)–(b); for part (a) $\sqrt{(33)}$ is between positive integers 5 and 6, for part (b) $x \approx 28$. Scholar provides an explanation that includes solid reasoning related to rational approximation.
Assessment 2 **8.NS.A.2**	Scholar does not correctly identify any of the greater numbers.	Scholar correctly identifies one of the greater numbers.	Scholar correctly identifies two of the greater numbers.	Scholar correctly identifies all three of the greater numbers.
Assessment 3 **8.NS.A.2**	Scholar makes little or no attempt to do the problem. OR Scholar may or may not place $\sqrt{5}$ correctly on the number line for part (a). For parts (b)–(c),	Scholar may or may not have placed $\sqrt{5}$ correctly on the number line for part (a). For parts (b)–(c), scholar may or may not provide a weak explanation for how the	Scholar places $\sqrt{5}$ correctly on the number line for part (a). For parts (b)– (c), scholar may provide a weak explanation for how the	Scholar correctly places $\sqrt{5}$ between 2.2 and 2.3 on the number line for part (a). For parts (b)–(c), scholar explains the method of rational approximation to locate the approximate

A Progression Toward Mastery				
	scholar does not provide an explanation.	number was placed. Scholar may or may not provide a weak explanation for how to improve accuracy of approximation.	number was placed. Scholar may provide a weak explanation for how to improve accuracy of approximation. Scholar references the method of rational approximation.	position of $\sqrt{5}$ on the number line. Scholar explains how to continue the rational approximation to include increasing smaller intervals to improve the accuracy of the estimate.