Grade Level	Content Area	Course Title *(9-12 Samples Only)*	Unit Title
Kindergarten	**Mathematics**	**N/A**	**Memorization Strip Boards—Addition & Subtraction**

Length of Unit	Time of Year Taught	Expected Prior Knowledge
Lesson time = 20-30 minutes *Approx. 23 lessons*	February-March through the end of the year *Note: Memorization work continues into the following year*	✓ Ability to actively attend (sit without disturbing others) a 20-30 presentation/lesson ✓ Ability to write digits 0-9 with adequate legibility ✓ Understanding of word-symbol-quantity to 20 ✓ Some previous concrete practice/lessons in addition as combining quantities and subtraction as taking away from quantities

Unit Description

One of the essential skills for students to master in the area of Operations and Algebraic Thinking is memorization of basic facts in addition, subtraction, multiplication and division. We believe that memorization is much easier to master when imbedded in concrete and multi-sensorial contexts. In this manner, students are able to construct mental representations from the concrete model. Prior to memorizing, students develop an inherent understanding of each operational process. This unit of instruction is one example of a series of lessons & presentations designed to elicit mastery in addition and subtraction operations by having students represent and articulate addition and subtraction processes using concrete materials/objects *(The Memorization Strip Board).*

Memorization means 'to learn something by heart'. It is not interesting by itself. At this point the teacher must choose either to force the children to memorize (not interesting to children who often try to avoid it), or to organize experiences with the children, all of which have the same goals—memorization and internalization of operational processes. In Montessori math, this is the concept of *parallel exercises* (memorization and operations practice with variety). Specifically, memorization is more interesting to children if it is reached through a range of different exercises.

Children in this age group have been presented with the entire preliminary work involving concept of quantity, place value, the importance of zero, etc. through presentations and practice with Montessori materials (Golden Beads, Stamp Game, and the Bead Cabinet). The points of consciousness with the *Golden Bead Material* is technique (systematic procedure), and an understanding of the meaning of each operation. With the *Golden Bead Material* the children receive a general overview of operations in the most concrete form—a 1:1 correspondence for quantity; the Bead Cabinet is also a 1:1 correspondence which aids in counting great quantities and recognizing fundamental patterns (i.e., making squares out of numbers); and the Stamp Game moves the child closer to abstraction by substituting a symbol (tile) for actual quantity. With the memorization material, the children get details on all the possibilities of the operations, especially with the two addends within the first nine digits.

With the above activities combined, the child learns that she/he can do any kind of addition just by knowing two combinations, and that subtraction has a unique relationship with addition (i.e., recognizing the pattern of 'fact families'). The child also understands that with little effort, they can do a lot of work!

The child has already worked with various concrete materials, such as the *Golden Bead Material*, the *Stamp Game*, and the *Bead Cabinet*. At a certain point, however, the child needs to learn the combinations in order to go on with her/his work. Learning is born out of interest and necessity--

when the child had to learn culture, she/he first had to learn to read. Likewise, to enter the world of mathematics, the child first must memorize, and so this process should be made as interesting as possible.

Memorization is necessary to lead children to abstraction—that is—to work without tables and actual quantities. For example, if given the operation 13 + 9, without memorization, most children would have to use their fingers, beads, drawings, etc. But with memorization, the operation is obviously much quicker. This series of lessons strengthens memorization by deepening the student's internalization and understanding of the processes of addition and subtraction. Once this is done, they can proceed with work at higher levels of difficulty in the area of operations and algebraic thinking.

Standards:	**Domain:** Operations and Algebraic Thinking **Cluster:** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. ***Standard #1:*** *Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.*
Summative Assessment	Random selections from the addition/subtraction Operations Booklets and cut combinations are assessed for meeting this standard. Students will represent and solve the selected equations, using the Addition and Subtraction Strip Board material to model each equation. **Criteria for each series of lessons in this sample:** **Lesson 1:** Using the Addition Strip Board, student will demonstrate/represent 3 random addition equations selected from the cut combinations 2-9 at 80% accuracy, or better. **Lesson 2:** Using the Addition Strip Board, student will demonstrate/represent 3 random addition equations selected from the cut combinations in lessons 2-13 at 80% accuracy, or better. **Lesson 14:** Using the Subtraction Strip Board, student will demonstrate/represent 3 random subtraction equations selected from the subtraction booklet at 80% accuracy, or better. **Lesson 15-23:** Using the Subtraction Strip Board, student will demonstrate/represent 3 random subtraction equations selected from the subtraction cut combinations box at 80% accuracy, or better. **Scoring of progress and mastery will also be recorded in the MRX system under the appropriate standard and area of instruction for the student.**

Scoring for Summative Assessment

Assessment & Scoring Rubric samples for addition and subtraction mastery of identified core standard:

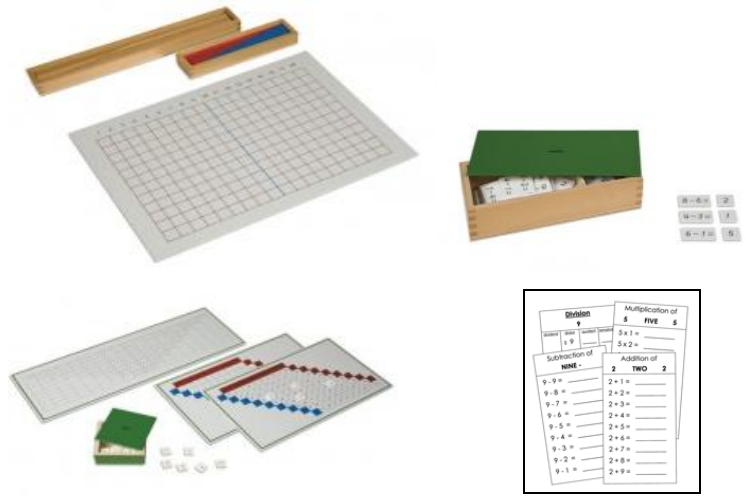
Student:			
Lessons 2-13: Using the *Addition Strip Board*, student will *demonstrate/represent 3 random* ***addition equations*** selected from the cut combinations in lessons 2-13 at ***80% accuracy, or better***. *(Record score & include observation notes in applicable boxes)*			
SCORING KEY: **1 = Needs more practice** **2 = Meets the objective** **3 = Exceeds the objective** ***Criteria for mastery = 11 pts or better***	**Date:** **Equation:**	**Date:** **Equation:**	**Date:** **Equation:**
9. *Reads* the equation aloud accurately			
10. *Forms* the equation correctly on the strip board			
11. Able to *explain the addition process* as putting together quantities			
12. Equation is *solved correctly*			
13. Equation is *recorded correctly*			
14. Indentifies/defines the *addend* as part of the total			
15. Identifies/defines the *sum* as the total from putting all parts together			
	TOTAL POINTS =	TOTAL POINTS =	TOTAL POINTS =

Student:			
Lesson 15-23: Using the *Subtraction Strip Board*, student will *demonstrate/represent 3 random* ***subtraction equations*** selected from the subtraction cut combinations box at ***80% accuracy, or better***. *(Record score & include observation notes in applicable boxes)*			
SCORING KEY: **1 = Needs more practice** **2 = Meets the objective** **3 = Exceeds the objective** ***Criteria for mastery = 13 pts or better***	**Date:** **Equation:**	**Date:** **Equation:**	**Date:** **Equation:**
1. *Reads* the equation aloud accurately			
2. *Forms* the equation correctly on the strip board			
3. Able to *explain the subtraction process* as taking away from quantities			
4. Equation is *solved correctly*			
5. Equation is *recorded correctly*			
6. Indentifies/defines the *minuend* as the total quanity			
7. Identifies/defines the *subtrahend* as the quantity to be taken away from the total			
8. Identifies/defines the *difference* as the quantity remaining (the answer)			
	TOTAL POINTS =	TOTAL POINTS =	TOTAL POINTS =

Materials/ Resources Needed	**Montessori Materials:** Addition & Subtraction Strip Boards Addition & Subtraction Table Booklets Cut Combinations Box—addition and subtraction	Number (answers) Tiles Addition (81 total equations) & Subtraction (90 total equations) Working Charts (includes 'control of error' charts)	**Other Materials:** Cut slips of paper for recording (i.e., 2 ¼ x 11 in.) White Board	Demonstration work space Dry erase markers Pencils and erasers

Addition & Subtraction Facts Memorization Materials

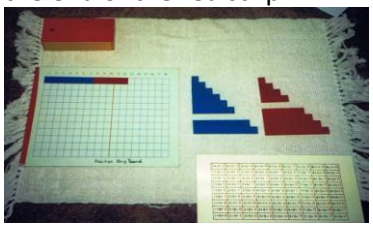
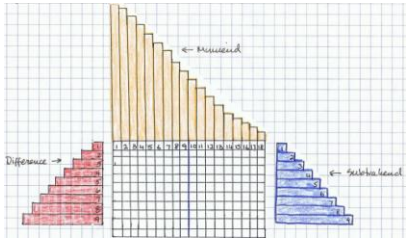
ADDITION

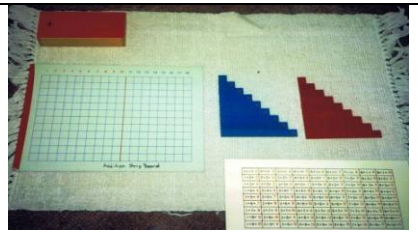
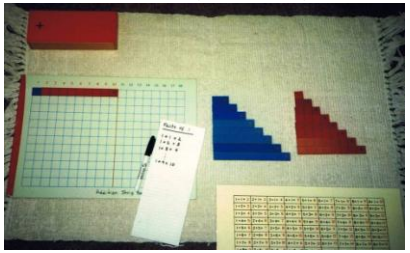


SUBTRACTION

Student copies of Addition/Subtraction Booklets

	Lesson 1—Addition Strip Board *Addition, Tables, Sequenced w/Booklets*	**Lessons 2-13—Addition Strip Board** *Addition, Tables, Random*	**Lesson 14—Subtraction Strip Board** *Subtraction, Tables, Sequenced w/Booklets*	**Lessons 15-23—Subtraction Strip Board** *Subtraction, Tables, Random*
Instruction and Timeline	**Aims:** ✓ *To provide the student with initial sensorial experience of the addition facts* ✓ *To provide practice in constructing addition equations with the Addition Strip Board* ✓ *To offer the student a chance to discover these facts independently* ✓ *To promote memorization of*	**Aims:** ✓ *Same as Lesson 1, however a closer step to abstraction with child recording equations.* In this presentation the Cut Combinations Box is used to randomly select equations. Take out a combination, read it and record the operation on a prepared slip (i.e., 8 + 2). Then	**Aims:** ✓ *To promote child's memorization of all of the possible combinations up to the equation 18 - 9.* ✓ *To provide practice in constructing subtraction equations with the Subtraction Strip Board* The Subtraction Memorization Boards present a variety of activities to reinforce basic subtraction facts up to 18.	**Aims:** ✓ *Same as Lesson 3, however a step closer to abstraction with child recording equations.* In this presentation, the Cut Combination Box is used to randomly select equations.

	these facts Begin by introducing the material to the child. Start with a sample equation, e.g., 7 + 5. Take the blue 7 strip (first addend) and place it on the first horizontal line of the strip board. Then take the red 5 strip (second addend) and place it to the right of the 7 strip on the same line. The result (sum) of the operation is above the end of the red strip.  This exercise leads the child to understand all the possible combinations necessary for memorization. Before presenting the booklet, lay out the material in front of the student: the blue strips in a triangular shape (with the '1' strip at top) and the red strips in the same manner to the right of the blue ones.	do the operation with the blue 8 strip (addend) and the red 2 strip (addend). Record the sum on the slip. After this short demonstration, the child can go on writing down and working combinations. The child can use Addition Chart #1 for 'control of error'. Eventually the child will be able to do the combinations without using strips. Lesson 1 & 2 in this sample are the beginning in a series of 13 separate exercises for the Addition Memorization Board. The remaining lessons are: ✓ Addition Strip Board (ASB)—tables--"commutative law" ✓ ASB—tables--"commutative law" w/zero ✓ ASB—tables--addend doubled ✓ Addition Working Chart #3 (AWC 3)—tables--complete table of addition ✓ AWC4—tables, "commutative law" revisited ✓ AWC5—tables, even and odd numbers	 Begin by introducing the material to the child. Point out that the Subtraction Strip Board is similar to the one for addition (one difference—the black line is after the 9 because the highest difference possible on the board is 9). Next, review the strips. There are 17 'neutral' strips which are used to cover the numbers along the top of the board (to establish the size of the first number in our equations, the "minuend"). Demonstrate how to line up the neutral strips. Then point out that the blue strips are used for the number we are taking away, the "subtrahend". Set these strips up to the right of the board. Next, do a sample equation to demonstrate how to use the material (i.e., 13-7). The minuend is 13, so you cover 14-18 on the strip board with the appropriate neutral strip. Then place the blue 7 strip to the left and next to the neutral strip. The	Have the child choose a combination (i.e., 12 – 8). Have the child record it on a prepared slip of paper. Put a neutral strip down to cover the numbers 13 through 18. Then put a blue 8 strip next to the neutral strip. The child can read the answer at the opposite end of the blue 8 strip (4), and then record it in the copybook. Continue this process with other combinations. Subtraction Working Chart 1 is the 'control of error', if needed. Lesson 1 & 2 in this sample are the beginning in a series of 9 separate exercises for the Subtraction Memorization Board. The remaining lessons are: ✓ Subtraction Strip Board (SSB)—tables—sequenced w/booklets ✓ SSB—tables—random ✓ SSB—tables—

	 Consider first the combination in the booklet, 1 + 1. Take the blue 1 strip and the red 1 strip and put them on the board. Then record the answer. Put away only the red 1 strip and keep the blue 1 strip on the table, since this lesson is a presentation of the combinations of 1. Now add the red 2 strip. Read the result and record the sum (1 + 2 = 3).  Repeat the process with 3, 4, etc., up to the last combination of 1 + 9 = 10. The teacher does all of the operations for this first exercise, allowing the child to take over if it seems appropriate. It is important to present this	✓ AWC6—tables, complete table ✓ AWC6—tables, answers first ✓ AWC6—tables, duplicate answers ✓ AWC 1—tables--missing addends ✓ AWC 1—Beginning story problems *Presentations and follow-up are the same as described in Lessons 1-2.* *As mentioned in the unit overview, Memorization Board work is just one of a variety of materials/exercises designed to strengthen memorization of facts, as well as strengthen understanding of operations processes. Therefore, the child may be working with other math materials simultaneously while working with Memorization Boards.*	difference, 6, can be read at the left end of the blue strip. Check for understanding, and then review the Subtraction Table Booklet. Begin with the '18 – 9' page. The minuend is 18, and the limit of the board is 18, so consequently no neutral strip is needed. Have the child place the blue 9 strip over the numbers from 18, and read the difference at the opposite (left) end of the blue strip, 9. Then have the child record the answer in the booklet. Next, turn to the page that begins with 17 – 9, and repeat the process. Then put the blue 9 strip back, and bring down the blue 8 strip to answer the second equation, 17 – 8. This completes the necessary memorization for combinations of 17. The booklet pages decrease from 18 and the possible number of combinations per page increases—that is, from 18 to 10. Then, after 10, there is a decrease in combinations down to 1 – 1. For example, for 18 there is only one combination to be memorized (18 – 9); for 17, there are two (17 – 9 and 17 – 8); and so on.	breakdown from zero ✓ Subtraction Working Chart 2 (SWC2)—tables—random ✓ SWC 3—tables—complete table ✓ SWC 3—answers first ✓ SWC 1—tables—missing digits ✓ SWC 1—Beginning story problems *Presentations and follow-up are the same as described in Lessons 1-2.*

	exercise in a way that does not bore the child. If it is necessary to skip around on the combinations in order to keep interest, then do so. With this presentation, Addition Working Chart #1 is used for the 'control of error'. It lists all of the possible combinations and their answers. Review how to use this chart with the child.		When the child has completed the booklet, it is time to present the Subtraction Working Chart 1. This chart has all of the possible subtraction combinations up to 18 – 1. With this control, the child may now check the answers recorded in their booklet. **Teacher's note:** On the left side of the table, all of the results are equal on each *horizontal line*, and on the right side of the table all of the results are equal on *the diagonal*. Do not point this out to the child, yet. It is very likely they will discover this as they continue with this series of lessons.	

	Lesson 1—Addition Strip Board *Addition, Tables, Sequenced w/Booklets*	**Lessons 2-13—Addition Strip Board** *Addition, Tables, Random*	**Lesson 14—Subtraction Strip Board** *Subtraction, Tables, Sequenced w/Booklets*	**Lessons 15-23—Subtraction Strip Board** *Subtraction, Tables, Random*
Student Activities and Timeline	Complete addition combinations for 2-9 (81 equations in all).	Complete 15-20 cut combinations before moving to the next lesson.	Complete subtraction table booklet for all combinations (90 equations in all).	Complete 15-20 cut combinations before moving to the next lesson.