Grade Level	Content Area	Course Title *(9-12 Samples Only)*	Unit Title
2nd Grade	**Mathematics**	**N/A**	**Word Problems: Comparative Subtraction**

Length of Unit	Time of Year Taught	Expected Prior Knowledge
1-2 weeks with extensions	Beginning of the year work--September	Understands addition and subtraction processes Extensive practice with addition memorization work Commutative Law (property) of addition Identity Property Word Problems—1st Level

Unit Description

David and Phyllis Whitin talk about the value of writing in the mathematics curriculum in chapter one of *Math Is Language Too: Talking and Writing in the Mathematics Classroom*. They state that mathematics and language are both "ways for learners to make sense of their world" and that ***"writing and talking are ways that learners can make their mathematical thinking visible."*** One of the most concrete examples of mathematics as language involves the reading and solving of word problems.

Word problems depend on reading comprehension skills for the development of successful problem-solving strategies. Having students collaborate on word problems gives them the opportunity to learn by talking, collaborating, and sharing ideas as they compare pictures, words, and numeric symbols for consistency. The Principles and Standards for School Mathematics, by The National Council of Teachers of Mathematics, includes communication as a mathematics tool for all levels of learners and suggests collaboration as especially beneficial for young learners.

Story and word problems are an essential part of the Operations & Algebraic Thinking domain of the Common Core Standards. In order for students to gain mastery in solving all types of word problem structures it is imperative they have a strong sense of number, quantity, and operational processes. In addition to the mathematical aspects and objectives, students must be able to comprehend the text of the story problem. Once students have completed most of the memorization work using various concrete materials to represent and construct basic equations they are ready to move on to working with larger quantities and more complex word problems.

This particular sample of lessons focuses on *reading comprehension skills* as they apply to mathematics word problems, as well as to developing the necessary *written and verbal mathematics communication skills*. It is important for students to learn how to analyze, represent, and solve equations in all four operations, although this lesson sample addresses just addition & subtraction. Through repeated exposure to and practice of various word problem structures using concrete materials and objects, students have developed effective strategies that are easily transferred to being able to represent word problems using only drawings and/or text. This is only a glimpse of the lessons and instruction the student will receive in this area of math over the year's span. In order to ensure our instruction aligns with the required standard, as well as student mastery of the standard, we have developed a rubric template for assessing the most common word problem structures as the level dictated by the standard.

Although this lesson sample presents lessons & practice specific to *comparative subtraction,* throughout the year students will continue to receive similar lessons & instruction in other types of word problem structures. Students are more apt to master solving one and two step equations with

unknown variables if they are presented with a variety of problem-solving strategies. Students will explore *five meanings for the operation of subtraction* (counting, sets, number line, balance, and inverse of addition). Comparative subtraction extends the student's early understanding about counting, addition, and subtraction in the *take-away mode*. ***Students will investigate properties of subtraction, represent subtraction in objects and pictures, and record subtraction in both vertical notation and equations.*** They will create and solve word problems involving comparative subtraction. Students answer such questions as "How many more?" and "How many less?" Missing addend activities provide students with an experience in algebraic thinking.	
Standards:	**Domain:** Operations and Algebraic Thinking **Cluster:** Represent and solve problems involving addition and subtraction. ***Standard #1:*** *Use addition and subtraction within 100 to solve one- and two-step word problems involving situation of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.*
Summative Assessment	When given a 1- step or 2-step addition or subtraction word problem within 100 and w/missing variables, the student is able to represent the equation by providing an accurate drawing/illustration which includes use of a symbol to represent the missing variable. Each of the 15 types of common word problem structures will be assessed for progress & mastery on the following rubric components: ▪ Information is presented in a logical sequence ▪ Drawing/illustration accurately represents the word problem ▪ Student uses "?", or "x", or other symbol to represent unknown variable ▪ Representation depicts all steps within word problem ▪ Student is able to verbally articulate the process and solution
Scoring for Summative Assessment	The teacher will complete the ***Word Problem Rubric*** for each word problem structure observed (see sample below). ***The criteria for mastery*** is 120 out of 150 points possible across 15 common word problem structures assessed on this rubric and at this level. Teacher will also input lesson sequence, student outcomes progress, and teacher observation notes into MRX data system.

Student Progress for Addition/Subtraction Word Problem Structures (1- and 2-steps within 100)

STUDENT:

SCORING KEY:
1 = Needs more practice
2 = Meets objective
3 = Exceeds objective

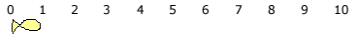
Criteria for mastery = +120/150 points (80%) for total 15 structures.

		Information is presented in logical sequence	Drawing/illustration accurately represents the word problem	Student uses "?", or "x", or other symbol to represent unknown variable	Representation depicts all steps to solution	Student is able to articulate how the solution was obtained	TOTAL POINTS	OBSERVATION NOTES
STRUCTURE A: "Add to"								
DATE:	1. Result Unknown **62 = 13 = _?_**							
DATE:	2. Change Unknown **62 + _?_ = 75**							
DATE:	3. Start Unknown **_?_ + 13 = 75**							
STRUCTURE B: "Take from"								
DATE:	1. Result Unknown **85 – 15 = _?_**							
DATE:	2. Change Unknown **85 - _?_ = 70**							
DATE:	3. Start Unknown **_?_ - 15 = 70**							
STRUCTURE C: Put together/Take apart								
DATE:	1. Total Unknown **65 + 35 = _?_**							
DATE:	2. Addend Unknown **35 + _?_ = 100**							
DATE:	3. Both Addends Unknown **Varies: 100 = 0 + 100; = 1 + 99; = 2 + 98; etc.**							
STRUCTURE D: Compare								
DATE:	1. Difference Unknown—"How many more?" **42 + _?_ = 65**							
DATE:	2. Difference Unknown—"How many fewer?" **65 – 23 = _?_**							
DATE:	3. Bigger Unknown—with "more" **42 + 23 = _?_**							
DATE:	4. Bigger Unknown—with "fewer" **23 + 42 = _?_**							
DATE:	5. Smaller Unknown—with "more" **65 – 23 = _?_**							
DATE:	6. Smaller Unknown—with "fewer" **_?_ + 23 = 65**							

Materials/Resources Needed	**Books:** ✓ *Fish Eyes: A Book You Can Count On*, Lois Ehlert	✓ Fish crackers ("healthy foods" variety) ✓ ½ inch grid paper Adjustable	✓ Masking tape or chalk ✓ Pan balance with weights **Activity Sheets:**

	✓ *Swimmy*, by Leo Lionni **Supplies:** ✓ Colored pencils, paper ✓ 10 each red, blue plastic plates	spinner (e.g., on-line spinner with adjustable number of segments—NCTM website) ✓ Number cubes ✓ Red & blue yarn	✓ One Less/One More Activity Sheet Differences Activity Sheet ✓ Number Lines Activity Sheet ✓ Pan Balance Activity Sheet ✓ Fact Families Activity Sheet

	Lesson 1—Counting Back	**Lesson 2—How Many More?**	**Lesson 3—Hopping Backward to Solve Problems**	**Lesson 4—Balancing Equations** **Lesson 5—Fact Families**
Instruction and Timeline	**Timeline:** Aims: Students will: ✓ model numbers and write numerals to 10 to use to answer questions about the relationship of numbers/quantities ✓ subtract from numbers to 10 by counting back ✓ record differences in vertical and horizontal format ✓ recognize the symbols used in recording subtraction Students count back to compare plates of fish-shaped crackers, and then they record the comparison in *vertical* and *horizontal* formats. They apply their skills of reasoning and problem solving during this lesson in several ways. [*Because students have associated the word "more" with addition, the comparative approach to subtraction is typically more challenging for the students to understand.*] To open--provide each student with fish crackers, two paper plates, colored pencils, and grid paper. Read aloud *Fish Eyes: A Book You Can Count On*, or some other counting book.	**Timeline:** **Aims:** Students will: ✓ find differences by comparing sets ✓ review the terms "addend" and "difference" ✓ explore the effects of subtracting 0 and subtracting all ✓ ***record differences in pictures, words, and symbols*** Students write subtraction problems, model them with sets of fish crackers, and communicate their findings in words and pictures. They record differences in words and in symbols. The *identity property of addition* is reviewed in the context of comparing equal sets. Call seven students to the front of the room, and then roll a number cube to decide how many more will come to form a second group. Next, ask each group to form a line so that the lines are parallel and the last	**Timeline:** **Aims:** ✓ Students will: ✓ use the number line model to find differences by comparing lengths ✓ solve puzzles using the number line In this lesson, students determine differences using the number line to compare lengths. Because this meaning is based on linear measurement, it is a distinctly different representation from the meanings presented in Lessons One and Two. At the end of the lesson, the students use reasoning and problem solving to predict differences and to answer puzzles involving subtraction. Using chalk or masking tape, make a number line on the floor. (The students will use this to find differences on a number line by hopping from a number toward 0.) Tell the students that they will now use the number line to compare lengths. Ask one student to hop to 5 and another to hop to 3. Then ask,	**Lesson 4:** This lesson encourages the students to explore another meaning for operations of subtraction, the balance. This meaning leads naturally into recording with equations. The students will imitate the action of a pan balance and record the modeled subtraction facts in equation form. **Aims:** Students will: ✓ model the balance meaning of subtraction ✓ ***draw/record the subtraction modeled/represented on the balance*** **Lesson 5:** **Timeline:** **Aims:** Students will: ✓ ***find missing addends*** ✓ review the additive identity ✓ ***generate fact families given two addends or given one addend and the sum*** In this lesson, the relation of addition to subtraction is explored with fish-shaped crackers. The

	As students listen, they will model each number named by adding one fish to a plate, recording the number at the beginning of the appropriate row on their grid paper, and finally placing the corresponding number of fish per square for the number row. Once finished with the story, have them remove one row of fish and color in the row (one row at a time). Have students color each row a different color (make sure they have 9-10 colors—white would be left uncolored). To the tune of an easily-remembered counting song (e.g., "A Hundred Bottles of Beer on the Wall") sing "Ten Little Fish on the Plate." Have the students place crackers on the appropriate bar on the graph they created in the activity above as they sing each verse of "Ten Little Fish." [This activity allows you to determine whether any students have trouble matching numerals to sets, counting, or writing numerals.] Now ask questions about their graphs, for example, *How many more squares are colored in the yellow bar than in the purple bar? How do you know?* [Possible answer: 1; I counted one more. This question introduces a *new mode of subtraction*—the *comparative mode*.] Next hold up a number card, such as eight, and ask the students to put the same	person in each line is standing against the board. Then tell each student to hold hands with the student across from him or her. Have a volunteer say how many more students were in the larger group, and then record the subtraction equation where all the students can see it. Continue the lesson by reading a book that features fish, such as *Swimmy*, by Leo Lionni. To help the students become more familiar with the set meaning for comparison subtraction, tell the students you are going to compare sets of crackers. Show a plate of fish-shaped crackers. Then roll a number cube and ask how many there will be in a plate with that many more fish-shaped crackers. Make a second plate with that many crackers to verify the students' responses. Then encourage them to write the subtraction equation that would be used to compare the two sets. Repeat this procedure several times. Then, if necessary, review the terms "addend," "compare," and "difference." Ask what the addends and difference would be if one plate has four crackers and the other has six? [4, 6, 2] Ask what the addends and difference would be if both plates had seven fish-shaped crackers [7, 7, 0]. Prompt the students to create other such entries. Next, ask the students to watch	"Who hopped farther? How much farther?" Repeat with other students. Next, draw a number line with the spaces one cracker apart, draw a red ring and place 3 fish-shaped crackers and a blue ring with 2 fish-shaped crackers inside. Ask: How many more fish-shaped crackers are in the ring with 3 fish-shaped crackers? How can we find out using the number line? Diagram 1: Number line with numerals the distance of one fish apart  Diagram 2: Circle with red fish and circle with blue fish Encourage the students to line up the crackers from the red ring with the left end of the number line. Then ask them to place the crackers from the blue ring in a line below the first line. Diagram 3: Number line with fish Next show how to hop back from the end of the longer line, counting the hops aloud. Have the students record the comparison using the equation notation [3 - 2 = 1] on the Differences Activity Sheet.	students search for related addition and subtraction facts for a given number and also investigate fact families when one addend or the difference is 0. Name two numbers, each less than 10. Then call on a volunteer to make sets of fish-shaped crackers corresponding to the numbers. It may be helpful to make the sets on different colored plates or to surround them with different colored yarn loops. Next have the student write as many addition sentences as he or she can. (If the numbers are not the same, two addition sentences are possible. For example: 1 + 9 = 10, 9 + 1 = 10] Then ask each student to use the sum and each addend to write as many subtraction sentences as they can. [In the example, the subtraction sentences would be 10 – 1 = 9; 10 – 9 = 1.) Name a difference (for example, 4) and have each student show you the meaning of the operations for the subtraction sentence by making two sets of crackers so that one set has four more crackers than the other. [There will be several answers to this question. Encourage the students to find as many ways as they can. Some pairs will be 6 and 2, 3 and 7, and 4 and 8.] Repeat with other differences, including 0. Next, have the students put six

	number of fish on a plate and record the number of crackers. Ask them to make a second plate with one less cracker. Now tell them they will compare the number of crackers on each plate. To do this, have them remove pairs of crackers from the plates by taking one cracker in each hand from each of the plates and placing the crackers in the bag until crackers are left on only one plate. Then have them count the crackers left on the plate. Ask for a volunteer to model and record the comparison using an equation in the horizontal format (for example, 8 – 7 = 1.); then record the comparison using the vertical format. Encourage the students to describe the comparison in at least two ways, in one instance using the word "more" and in the other instance using the word "less." [For example, “There is one more cracker on the plate that had eight crackers.” “There is one less cracker on the plate with seven crackers.”] For the next part of the lesson, students will need access to the *adjustable spinner*. Ask a student to choose a number from 5 to 10. Divide the spinner into that many parts. Now call on a volunteer to spin the spinner and call out the number that comes up. Ask the students to model the number by putting that amount on a plate. Have them show a group of fish that is *one more than* that number, and then a group that is *one less*. You may wish to have them record their responses on the *One Less/One More Activity Sheet.* Repeat with other students.	as you solve a subtraction problem in which two sets are compared. For example, you could say that Jen’s plate has five crackers and Sally’s plate has three crackers. Then create two sets where everyone can see them, surrounding one set with red yarn and the other with blue yarn. Ask the students to imagine the yarn loops represent plates. (If you prefer, you can use red and blue plates for the demonstration and for the student activity.) Then ask questions such as the following: ✓ What comparing questions can we ask and answer about the plates? ✓ How many more crackers are on (student's name) plate? ✓ How many fewer crackers are on (another student's name) plate? Now give each student two lengths of yarn and some crackers. Have the students pose comparison situations, model them, and answer similar questions. When the students are ready, present a subtraction story problem in which a set of three and a set of four are compared. Demonstrate how to make a horizontal bar graph that will allow the students to compare the data. Next, guide the students through the solution of	It is not uncommon for the students to count the lines on the number line rather than the spaces covered by the hops. You may wish to highlight the fact that in this meaning for the operation of subtraction, spaces are counted, not points on the number line. You may demonstrate this by using a length of paper the size of a fish-shaped cracker to hop back with. After several examples, show the students that they do not need to place the crackers themselves on the number line, but can mark the length with a crayon. To enrich the students' understanding of the number line concept, model how to use the Number Line Arithmetic Tool from the National Library of Virtual Manipulatives to compare lengths. Encourage the students to use this site during math center time, and assign students to work at the site in pairs. Those not taking their turn at the computer should complete the next activities. Put the students into pairs and give each pair fish-shaped crackers, crayons, and one number line from the Number Lines activity sheet. Ask each student to make two sets of crackers on a piece of paper, and then enclose each in rings of different colors. Then	crackers on one plate and two on the other. Ask them to describe the relationship between the plates using a subtraction sentence and the related addition sentences. (For example, 6 - 2 = 4; 4 + 2 = 6; 2 + 4 = 6) When the students are ready, ask them to identify the addends, the sum, and the difference. Then ask what subtraction sentence can be made with the sum and the other addend. Tell them that this set of equations is called a fact family. Then assign the students to pairs, provide them with the Fact Families activity sheet, and have them take turns making plates, recording the sums and addends, and writing equations. After the students have recorded several fact families, be sure all the students have additional experiences with the additive identity by asking the students to compare two plates, one with zero fish-shaped crackers on it and one with seven crackers, and to write the related addition and subtraction sentences. [The equations will be 7 - 0 = 7; 0 + 7 = 7 ; 7 + 0 = 7] Now ask the students to make two plates each with seven crackers on them, then record the comparison and the two addition sentences. [The equations this time will be 7 - 7 = 0; 0 + 7 = 7; 7 + 0 = 7].

	To end the lesson, have the students divide some crackers between two plates, then remove a cracker from each plate until all the crackers on one plate are gone. **Questions for Students** ▪ How many fish-shaped crackers are on this plate? (Show a plate with six crackers.) On this plate? (Show a plate with seven crackers.) Which plate has more? How many more? Which plate has less? How many less? How can you prove that? ▪ [6; 7; the second plate has more; 1 more; the first plate has less; 1 less; student responses may vary.] ▪ What number sentence would show that you compared a plate of six fish-shaped crackers with a plate holding five fish-shaped crackers? ▪ [6 - 5 = 1] ▪ What is the same in the two ways that we recorded the comparisons? What is different? ▪ [The same numbers are compared, but the way we wrote them was different (one way was horizontal, and one way was vertical.)] ▪ How could you help a younger student find the answer to 7 - 6 by counting? ▪ [Student responses may vary.] ▪ What equation shows that you compared a plate of 8 fish-shaped crackers with a plate with 10 fish-shaped crackers? ▪ [10 - 8 = 2] ▪ What does the minus sign mean? ▪ [Subtract, or take away]	another problem, this one showing the comparison of a plate of two crackers with a plate of three crackers. (For example, Meg had three fish and Pete had two fish. How many more fish did Meg have?) Then ask the students to record the sets in a bar graph. When they are ready, call on the students to share their problems and the graph. You may wish to suggest that they record a comparison in pictures, as a bar graph, and in an equation for their portfolios. **Questions for Students** ▪ What do we find when we compare two sets? ▪ [The difference.] ▪ Make two plates of crackers. Can you show how to compare the two plates? ▪ [Student responses may vary, but they should be able to identify the major concepts covered in today's lesson.] ▪ If we compare a set of five and a set of eight, what will the difference be? How would you verify that? ▪ [3; 8 - 5 = 3; students may use a bar graph, subtract, draw two plates, etc.] ▪ What would be the smallest	have the students line up the crackers carefully and draw, in the appropriate colors, a line as long as the number of crackers in the set. Then ask them to compare the lengths on the number line to find the difference and to record the comparison in pictures and in equation form. After allowing time for exploration, call the students together to read their equations and share their number line illustrations. As a concluding activity, pose puzzles such as "I am thinking of two numbers on the number line that have a difference of 5. The larger number is 6. What is the other number?" (If the students are ready for a challenge, you might say only: "I am thinking of two numbers on the number line that have a difference of 5. What are the numbers?") You may wish to have the students create and share similar problems. One or more of these puzzles could be added to their learning portfolios. **Questions for Students** ▪ How could you use the number line to compare two plates, one of which has five fish-shaped crackers and the other of which has three fish-shaped crackers? ▪ [Draw a number line, mark the places for five and three with fish, and then compare the distance between them.]	Then call the class together, and ask a volunteer to choose one row from the Fact Families activity sheet and demonstrate what the two plates would look like. Ask his or her partner to write the number sentences that the plates suggest. You may wish to repeat this with other volunteer pairs. Now invite one of the students to make two plates, each one with three fish-shaped crackers on it, and write the related addition and subtraction sentences in the 3, 3, 6 family. [There will be only one of each, 3 - 3 = 0; 6 - 3 = 3.] Finally, ask the students to record a set of number sentences about one of the rows they completed and illustrate it by drawing pictures of two plates that they made. **Questions for Students** ▪ If you know that one plate has eight fish-shaped crackers on it and another has three fish-shaped crackers on it, how many more are in the plate with eight crackers? ▪ How many addition and subtraction facts can you write if you compare a plate holding three crackers and one holding five crackers? How are the facts alike? How are they different? ▪ Suppose I make a plate with four fish-shaped crackers and a second plate with four fish-shaped crackers. What sentences will describe a

	▪ What two symbols did you use to mean "equal to"? ▪ [In the horizontal equation form '=' is used. In the vertical form the line means "equals."] ▪ When you count backward, what comes after 10? After 7? After 1? What comes before 6? Before 4?	difference we could get between two plates if one plate has four fish-shaped crackers? How will I get that difference? ▪ [0; Put four crackers on each pate] ▪ Suppose you had a plate of five fish-shaped crackers. How many crackers would you put on a second plate so that there would be a difference of 4 with that plate? Is there another way? ▪ [There are two ways--a plate of one and a plate of nine.] ▪ Look at one of the bar charts. How could we act it out with lines of students? With sets of crackers	▪ What numbers have a difference of 2? Can you find some of them on the number line? ▪ [Some examples include: 5 - 3, 4 - 2, and 3 - 1] ▪ What would be the difference if two plates had the same number of crackers on them? Would the lines that showed how many crackers are in each plate be the same length? How do you know? ▪ [0; yes; student responses may vary.] ▪ How would you explain to a friend how to compare lengths on the number line? ▪ [Student responses may vary.]	comparison of the plates? ▪ How could you help a friend find addition sentences related to 5 - 2 = 3? To 4 – 0 = 4? ▪ What addend pairs can you find for a difference of 2? What subtraction sentences do they suggest? How would you model the comparison? ***Original source for this unit: www.readwritethink.org***

	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4-6**
Student Activities and Timeline	2nd period practice: Put the above questions on cards, place on the math shelf and have students pull and complete a negotiated number of problems per week.	2nd period practice: Put the above questions on cards, place on the math shelf and have students pull and complete a negotiated number of problems per week.	Put the following **Challenge Question** to students—have them record/copy the question into their math notebooks, work on the solution, and finally, invite student to share "proof" with teacher. *If I have a blue pencil that is 3 inches long and a red pencil that is 5 inches long, which pencil is longer? How much longer is it? How could you use a number line to prove your answer?*	**[A Challenge Question]** Suppose I tell you that 7 is 3 more than some number. Can you write the number sentence that says that? [7 = 3 + _.] If I said 7 is 3 less than a number, what number sentence can you write? [7= _ - 3.] **[A Challenge Question]** How could you explain to a classmate how to find all the members of a fact family?
NOTE: 2nd period practice: Representing and solving one & two step word problems with missing variables (word problem cards a year-long assignment). Teacher should record progress/mastery (on the specified rubric) of any student work generated by these lessons & activities.				